A TEXT BOOK
OF
ASTRONOMY

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PREFACE

The book is intended to be of the standard of the B.A. and B.Sc. Pass Course of Calcutta and other Indian Universities. Nothing original is claimed for a work so elementary, and on so widely written a subject. We only hope the arrangement will be found to be logical and the treatment both brief and clear. Also, our aim throughout has been to provide the reader with clear fundamental notions of Astronomy rather than exhaustive information.

Though Stellar Astronomy is not explicitly included in the course in view, we have devoted a chapter to it at the end, dealing mainly with dynamical aspects. A companion book is in course of preparation, in which Spherical Trigonometry and generally more mathematical details will be introduced to meet the needs of Honours students.

We are perhaps unconsciously, but none the less surely, indebted to many excellent astronomical books of diverse nature, both by Indian and European authors. We acknowledge our general indebtedness to them all.

Our most cordial thanks are due to Book Society of India Ltd., and particularly to their Director, Prof. B. Banerjee who accomplished the feat, almost impossible under present conditions, of seeing the book through the press in the course of three weeks.

We shall be grateful to any one pointing out mistakes which probably cannot be avoided in so hurried a publication, and making helpful suggestions for improvement.

Calcutta,
July, 1948.

B. C. B.
M. C. C.

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CONTENTS

1. INTRODUCTION

1. 1. *Astronomy* is the science of celestial bodies. Originally it was chiefly concerned with the study of motion of 'planets'. In modern times different aspects of the study of stars occupy the centre of interest. It would be artificial to divide the subject into clear-cut branches. But in order to give an idea of the topics we shall discuss in the following pages, we may classify astronomical topics in general as (i) mathematical and (ii) physical. The object of the first class of topics is, for instance, to discover true motions of celestial bodies from apparent, to offer dynamical explanation for them, to explain phenomena (such as eclipses) which are the consequences of such motions; to determine distances, dimensions and masses, etc. of celestial bodies, and so on. On the other hand, the object of the second class of topics is to discover elements or any compounds in the composition of celestial bodies, to find out the temperature and the state of matter (*i.e.*, whether solid, liquid or gaseous) on the surface and if possible in the interior of celestial bodies, to explain the source of heat and light of the sun and other celestial bodies, and so on.

We shall be concerned chiefly with topics of the first class.

1. 2. We must start on our study of Astronomy with observation of apparent motion of celestial bodies. Suppose we take our stand in an open field, on a clear moonless night. The sky appears to be hemispherical, bedecked with apparently innumerable points of light, generally called stars. To get apparent motion of a star, its position must be observed at intervals. But it is obvious at once that of the two elements (i) direction and (ii) distance which define position, there is no immediate means of measuring the second. We therefore represent the position of a celestial body by the point in the same direction on a sphere having the observer at the centre. To represent all celestial bodies, the sphere must be complete. For, undoubtedly there are, at any instant, celestial bodies below the horizon; only we cannot see them on account of the intervention of the opaque matter of the earth.

Definition: The sphere described with the observer at centre, on which we choose to represent positions of all celestial bodies, is called the *Celestial Sphere:* such representation, it is to be understood, is only with regard to direction and not distance of a celestial body.

1. 3. Though we are unable at the very start to measure

distances of stars, it is possible to infer from quite simple observations that they must be enormous. Suppose we measure angles subtended at the eye by two particular stars S and S′, from various points of the earth's surface, A, B,...(Fig. 1.3). They are found to be all equal; even the finest instrument fails to detect any significant difference among them.

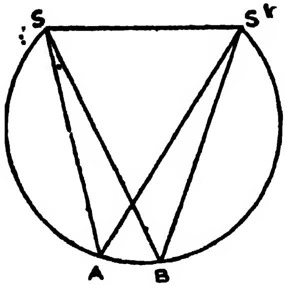

Fig. 1.3

If ∠ SAS′= ∠ SBS′ rigorously, A, B, .must lie on a circular arc SABS′. Hence the surface of the earth should be that formed by revolving the arc about SS′. Such a surface is concave but from observations, a familiar example of which is that first the hull of a ship and then the mast gradually disappear when the ship sails away from port, it is obvious that the earth's surface is convex. So ∠ SAS′, ∠ SBS′ .. are not rigorously equal. That they appear to be so must be because ∠ ASB, ∠ AS′B′,.. .. are immeasurably small.. Hence AS, AS′, BS, BS′ are incomparably greater than AB.

Stars being situated at distances compared with which distances on the surface of the earth are insignificant, we may look upon the radius of the celestial sphere as infinite and the entire earth as merely a point.

1. 4. Since in our study of Mathematical Astronomy, we have constantly to do with a sphere, we need a few definitions and simple theorems relating to points and circles on a sphere.

Theorem (a): Any plane section of a sphere is a circle which is the greatest when the plane passes through the centre of the sphere and the radius of this greatest circle is equal to the radius of the sphere.

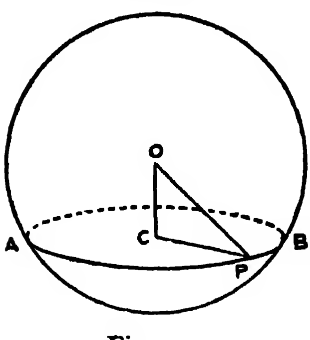

Fig. 1.4a

Let APB be a plane section of the sphere, centre O (Fig. 1 4a). Draw OC perpendicular to the plane meeting it at C. Take any point P on the section APB. Join CP, and OP. CP, being in the plane APB, is perpendicular to OC. Hence

$CP^2 = OP^2 - OC^2 = r^2 - p^2$ where r = the radius of the sphere and p = the perpendicular OC.

For different points P, r and p remain the same; hence the distance of any point P on APB from C is constant; in other words, the section APB is a circle, C is its centre, and its radius is equal to $\sqrt{r^2 - p^2}$. The section is the greatest when $p = 0$, *i.e.*, when its plane passes through the centre O, and its radius is then equal to r.

Definition : The greatest circles that can be drawn on a sphere are called *Great circles.*

We have seen above that the radius of a great circle is equal to that of the sphere.

Definition: All circles on a sphere, other than the greatest, that can be drawn, are called *Small circles.*

Definition: The perpendicular, drawn through the centre of a sphere, to the plane of a great circle, meets the sphere in two points called *Poles* with reference to the great circle. Conversely, with reference to the Poles the great circle is called the *Equator.*

Poles of a great circle may also be called Poles of any small circle whose plane is parallel to that of the great circle.

Definition: Small circles whose planes are parallel to that of a particular great circle are called simply *Parallels* with reference to it or to its poles.

Definition: Any great circle passing through the poles of another great circle, is called a *Secondary* to the latter.

In Fig. 1.4b, POP′ is perpendicular through O to the plane of the great circle ACB. The points P,P′ where it meets the sphere are therefore Poles of the great circle ACB; ACB is conversely the Equator with reference to P and P′. The great circle PCP′ passing through P and P′ is a Secondary to the great circle ACB. There is plainly an infinite number of Secondaries to the great circle ACB, because an infinite number of planes pass through P and P′.

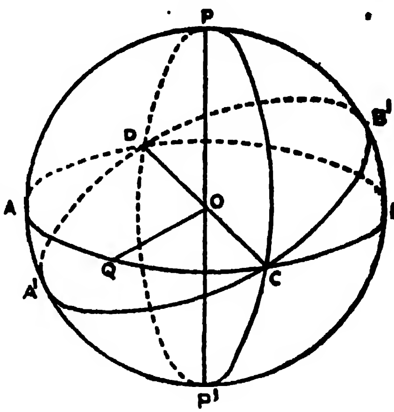

Fig. 1.4b

Theorem (b): One and only one great circle can be drawn through two given points on a sphere.

For, besides the two given points, the plane of the great circle must pass through the centre of the sphere. These three points determine one and only one plane and so one and only one great circle.

Theorem (c): The plane of a Secondary to a great circle is perpendicular to the plane of the great circle.

For the plane of the secondary contains both the pole and the centre of the great circle *i.e.*, a normal to the plane of the

great circle. In Fig. 1.4b, the plane of the Secondary PCP′ contains OP which is normal to the plane of the great circle ACB. The plane PCP′ is therefore perpendicular to the plane ACB.

Theorem (d): Two great circles bisect one another.

Let ACB and A′CB′ be two great circles and O the centre of the sphere (Fig. 1.4b). Their planes intersect in a straight line, and O, being a common point, lies on it The line of section COD is therefore a diameter to each circle. It follows that they bisect one another.

Theorem (e): If the pole of one great circle lies on another, the pole of the second lies on the first.

Let the pole P of the great circle ACB lie on the great circle PCP′ (Fig 1 4b) To prove that the pole Q of PCP′ lies on ACB

Since P is the pole of ACB, and O is the centre of the sphere, OP is perpendicular to the plane of ACB. Similarly OQ is perpendicular to the plane PCP′ and therefore perpendicular to OP which lies in PCP′. But all perpendiculars to OP through O must lie in ACB Hence OQ lies in ACB, *i.e.*, Q lies on the great circle ACB.

Cor. If one great circle be secondary to another, the latter is also a secondary to the former.

Theorem (f): The pole of a circle is equidistant from every point on the circumference of the circle.

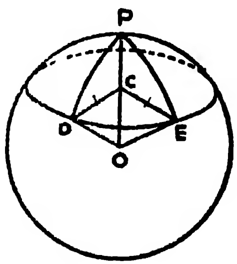

Fig. 1.4c

Let O be the centre of a sphere and ED a circle on it. Let P be the pole of the circle. Then OP is perpendicular to its plane. Let OP meet the plane of the circle in C; then C is the centre of the circle Take any two points D and E on the circle. Join CD, CE, OD, OE. From the identity of the two triangles OCD and OCE, $\angle$ POD = $\angle$ POE. Therefore arc PD = arc PE. $\therefore$ P is equidistant from D and E.

1. 5. *Definition:* When three points on a sphere are joined two by two, by arcs of great circles, each less than a semicircle, the figure formed is called a *Spherical triangle.*

Definition: The angle between two arcs of great circles is the angle between their planes.

The arcs joining the points are called the sides, and the angles between the arcs, the angles of the spherical triangle. It is usual to measure sides of a spherical triangle by the angles they subtend at the centre of the sphere. If the radius of the sphere be unity

and the angles subtended by the arcs be in radians, the measure of the arcs and of the angles subtended by them are the same.

Let ABC (Fig. 1.5b) be a spherical triangle. As

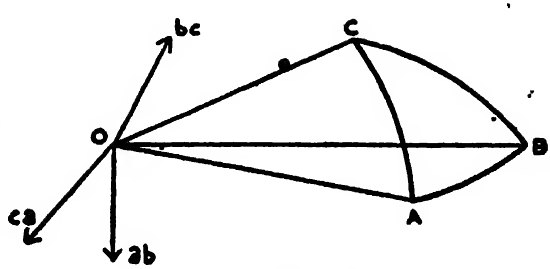

Fig. 1.5b

with plane triangles, the sides BC, CA, AB are denoted by a, b, c, and the angles at the opposite vertices by A, B, C; a, b, c, A, B, C are measured in radians.

Theorem (a): The sum of any two sides of a spherical triangle is greater than the third.

Let ABC be a spherical triangle (Fig. 1.5b). Join the vertices A, B, C to the centre of the sphere O. Then the face angles of the trihedral angle at O are a, b, c. It is proved in elementary geometry that the sum of any two face angles is greater than the third. Hence the theorem.

Theorems in plane geometry on the identity of two triangles have their analogues in spherical geometry. We do not establish them in detail, because such cases as may occur in our study will be more or less obvious. Generally, two spherical triangles are identical if they have the following elements equal:—(i) Three sides; (ii) three angles; (iii) two sides and included angle; (iv) two angles and adjacent side; (v) two sides and one opposite angle; (vi) two angles and one opposite side. Cases (v) and (vi) are ambiguous; but if we are sure from the conditions of a problem that there is no ambiguity they will be identical. A point of difference between spherical and plane triangles, worthy of notice, is the following:

Theorem (b): The sum of the angles of a spherical triangle is greater than two right angles.

Let ABC be a spherical triangle (Fig. 1.5b). Join OA, OB, OC and draw the outward normals to the planes OAB, OBC, OCA; let them be Oab, Obc, Oca. They form a trihedral angle at O; the sum of its face angles is less than four right angles. But it is easy to see that the face angles are $180° - A$, $180° - B$, $180° - C$. Hence $3 \times 180° - A - B - C < 360°$; or $A + B + C > 180°$.

Examples Worked Out

1. A great circle is drawn through the poles of two other great circles. Find the poles of the former.

Let the two great circles intersect at A and B, and let P and

Q be their poles. Join OP and OQ, where O is the centre of the sphere. OA is perpendicular to OP and also to OQ. Therefore OA is perpendicular to the plane of OP and OQ. Hence A is the pole of the great circle PQ. Similarly B is the other pole.

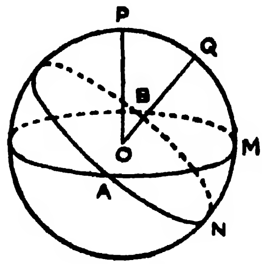

Fig. 1.5c

2. Show that:—(1) the angle between two great circles is the angle between their poles; (2) the angle between two great circles is the arc intercepted by them on their common secondary.

Let P and Q be the poles of the great circles. Then OP and OQ are respectively perpendicular to their planes; the angle POQ is the angle between the planes, *i.e.*, between the great circles.

Let the great circle PQ meet the two given great circles at M and N respectively. Then arc PM = arc QN, each being 90°. Take away the common part; ∴ arc MN = arc PQ = the angle between the great circles

3. If γM and γN be two given great circles and σ M and σN are two other great circles drawn perpendicular to them, will the angle M σ N be always equal to the angle M γN? Show that they will be equal if σN = γM.

Let P, K, m and n be the poles of γM, γN, σM and σN respectively (Fig. 2.4b). P and K will always lie on σ M and σ N and m and n will lie on γM and γN. Imagine γ M, γN and σ M to be fixed and the great circle σ N and so its pole n to be moving. Since the angle M σ N is equal to the angle between m and n it follows that it is variable and not always equal to the angle M γ N.

When σ N = γM the identity of the two right angled triangles is easily established and so the angle M σ N = the angle **MγN**.

Exercise 1

1. Give an idea of the subjects studied in Astronomy
2. How do you conclude that distances of stars must be enormous compared with distances on the surface of the earth?
3. Define Great circle, Small circle, Equator, Pole, Secondary, Parallels.
4. Can you draw a great circle through three points on a sphere? Can you draw a small circle through them? If so, how many?
5. Can you draw two great circles parallel to one another?
6. Find the angles and sides of the spherical triangle formed by joining the points where Oab, Obc, Oca meet the sphere in. Fig. 1.5b.
7. Show that A + B + C > 180° for a spherical triangle. Is the sum of the sides a, b, c greater or less than four right angles?
8. If a point be equidistant from three great circles, it is equidistant from their poles. How many such points are there?

2. THE CELESTIAL SPHERE

2. 1. As a preliminary, it is convenient to divide celestial bodies into groups from a general consideration of their apparent motions. Below we describe such motions, group by group, assuming that the reader can imagine devices by which they can be practically determined. Methods and instruments for more systematic and accurate observation will be described in a subsequent chapter.

STARS:- By far the largest number of celestial bodies, called stars, fall into a group. They retain the same configuration among themselves *i.e.*, they move as if bound together on a rigid frame. Other bodies exist, called planets, which have similar appearance to the naked eye. They can however be easily distinguished by the essential criterion of apparent motion. If a star-like body be found to change its relative position among stars in course of a few days, it is a planet. It is now possible to obtain photograph of any portion of the sky by a few hours' exposure. In such a photograph, if stars appear as points, a planet in the field of view, appears as a short line, on account of its motion, relative to the system of stars. One who is familiar with the aspect of the sky can detect a planet, on a mere inspection of the region; it strikes him as a new object in the familiar arrangement of stars. A ready means of distinguishing a planet from a star is provided by the fact that it appears as a disc in a telescope of even moderate magnifying power, while stars appear as points of light, however high the magnifying power may be. It is sometimes possible to distinguish stars from planets by the fact that stars shine with twinkling light and planets shine with steady light. A star twinkles *i.e.*, becomes alternately bright and faint due to the fact that the density of air is always changing and therefore light from a star undergoes constant changes in its passages through it to the observer. Planets do not twinkle because they are discs of light and not luminous points like stars. In reality every point of the disc of the planet twinkles like a star, but the different points do not do so in unison, so that the sum of light remains nearly uniform.

Common features in apparent motion of all stars are:

(1) Paths of stars are small circles round a common pole *i.e.*, the paths are parallels.

(2) The motion is uniform throughout.

(3) The period of a complete revolution in the path is the same for all stars and is 23 hr. 56 min. 4 sec. of civil time.

On account of their unchanging configuration, stars were

called *fixed stars* by the ancients. We shall however see later, in CHAPTER 15, that they do have individual motions, though they are very minute and are never appreciable to the eye even in centuries. But at preliminary stages of his study, the reader should regard stars to be fixed.

Definition: The points where the diameter of the celestial sphere, which is perpendicular to the parallel planes of the small circles daily described by stars, meet the celestial sphere are called *Celestial poles.* In other words, the common poles of the parallels, daily described by stars, are called celestial poles.

The pole in the northern hemisphere is called the *North Celestial pole* and that in the southern, the *South Celestial pole.*

Definition: The great circle whose plane is perpendicular to the diameter passing through the celestial poles, is called the *Celestial Equator.*

Motions of other celestial bodies are best given by their relative motions with respect to stars. We imagine the celestial sphere, with stars fixed on it, to be at rest. Relative motion is then motion against the fixed background of stars. Motion of stars, as well as of other celestial bodies, which is daily experienced by the observer, is of course reproduced by revolving the celestial sphere from east to west once in 23 hr. 56 min. 4 sec.

THE SUN: Relative to stars, the sun moves towards the east and describes a great circle in a period called the *sidereal year.* The motion is not strictly uniform; the nature of the variability will be given in a subsequent chapter. In consequence of this relative motion, the sun takes on an average 24 hr. of civil time to make a complete diurnal revolution, a little in excess of the period of revolution of a star.

Since there is no other celestial body which has the same characteristic motion, the sun by himself forms a group.

Perhaps the reader may find it difficult to see how the path of the sun among stars can be traced, when the latter are not visible at all in day time. The systematic method of recording positions of all celestial bodies will be explained later on. For the present, we may adopt the following plan. When the sun comes up to the meridian (for the definition of the term, see SEC. 2.3) note the time and the elevation above the horizon. The star, diametrically opposite the sun at the instant, comes up to the meridian exactly after half the period of diurnal rotation of stars. Its elevation, when on the meridian, can also be easily calculated. Having found out the star on the meridian, the position of the sun at his previous passage over the meridian, can be located on a star-map at the diametrically opposite point. Repeating the, observation day after day we can trace out the whole path after a year's observation.

Definition: The great circle described by the sun on the celestial sphere, relative to stars, is called the *Ecliptic*.

The name Ecliptic was given by the ancients, who noticed that an eclipse took place only when the moon is near this circle. The ecliptic cuts the celestial equator at an angle 23°28′ approximately. This angle is called the *obliquity of the Ecliptic*.

THE MOON: The moon also moves towards the east and describes a great circle on the celestial sphere, relative to stars. The period of a complete revolution of the circle is 27 ⅓ days, approximately. Other details of the moon's motion are given in a later chapter. It is however clear that the moon should be treated as a separate group by herself, because her motion is different from that of the sun or any other celestial body.

THE PLANETS: The motion of a planet, relative to stars, is curious at first sight. It moves among stars, at times towards the east, at times towards the west, and at times it is stationary among them. On account of this seemingly wayward motion, it has been called by the ancients 'planets', which literally means 'wanderers'.

COMETS: A comet has a distinctive form possessing a head and a comparatively large tail. It makes its appearance suddenly and remains in view generally for a short time. Its motion has features similar to that of the planets; but there are points of difference too.

METEORS. Meteors or shooting stars are celestial bodies and also a subject of our study. They flash into sight only for a few seconds during which all observation is to be made. The scanty information so gathered gives, as we shall see later on, a clue to their origin and nature.

2. 2. The ancients noticed that the sun, the moon, and the planets are always to be found within a limited belt of the celestial sphere bounded by two small circles, one on each side of the ecliptic and distant 8° from it. This belt is known as the *Zodiac* which means 'zone of the animals'. The name has been given by the ancients from imaginary resemblance of different constellations (*i.e.*, groups of stars) situated in the belt to different animals. The Zodiac is divided into twelve sections, each of 30°, marked by twelve constellations which are called the ***signs of the zodiac***. They are in order towards the east:

Aries	Taurus	Gemini	Cancer	Leo	Virgo	Libra
मेष	वृष	मिथुन	कर्कट	सिंह	कन्या	तुला

Scorpio	Sagittarius	Capricornus	Aquarius	Pisces
वृश्चिक	धनु	मकर	कुम्भ	मीन

Definitions: The points at which the ecliptic cuts the equator are called ***the First point of Aries*** and ***the First point of Libra***. The

First point of Aries (symbol γ) is the point on the equator through which the sun, in his motion relative to stars, crosses from the southern to the northern half of the celestial sphere. The First point of Libra (symbol Ω) is the point through which the sun crosses from the north to the south of the equator in his relative motion among stars. These names were given by the ancients when the points were actually situated in the respective constellations. They have since shifted backward and the point called the First point of Aries is now in the constellation Pisces, and that called the First point of Libra, in the constellation Virgo. Still the old names are retained.

The sun arrives at the First point of Aries every year on the 21st March; he is 90° farther on on the 21st June; at the First point of Libra on the 23rd. September and 90° farther on on the 21st December.

3. *Definition:* The direction of the plumb line is the *vertical direction*. The great circle on the celestial sphere whose plane is perpendicular to the vertical direction is called the *Celestial horizon* or simply *Horizon*

Definition: The vertical line drawn through the centre meets the celestial sphere in two points; the point above the horizon is called the *Zenith* and the point below the horizon the *Nadir*.

Definition: The great circle through the zenith and the celestial pole is called the *Celestial meridian* or simply the *Meridian*. Note that by Theorem (b), SEC. 1.4, only one such great circle can be drawn.

Definition: When a celestial body is just on the meridian it is said to be in *transit*.

The points where the meridian meets the horizon are called the *North* and the *South points*. The celestial equator cuts the horizon in two points which are called the *East* and the *West points*. It is easily seen that these points of intersection are 90° from the North or the South point. The poles of the horizon and the equator both lie on the meridian. Hence by Theorem (e), SEC. 1.4. the pole of the meridian lies on both the horizon and the equator; in other words, it is at the intersection of the two. Hence its distance from the north or the south point, both of which are points on the meridian, is 90°. Of the two points of intersection, the one to the right when we face north is the east point and the other, the west point. The north, south, east and west points are called the *Cardinal points* of the horizon.

Definition: Any great circle through the zenith, *i.e.*, any secondary to the horizon is called a *Vertical*. The vertical through

the east and the west points is called the *Prime vertical*.

In Fig. 2.3a, the straight line ZOZ′ is the vertical direction, Z is the zenith and Z′ the nadir. The great circle N E S W perpendicular to ZOZ′ is the celestial horizon. The great circle ZPN through the zenith Z and the celestial pole P is the meridian. The points N, S where it meets the horizon are the north and the south points. The celestial equator QEQ′ cuts the horizon at E and W which are the east and the west points. Great circles ZS_1M_1, $Z'S_2M_2$, ZS_3M_3 are verticals.

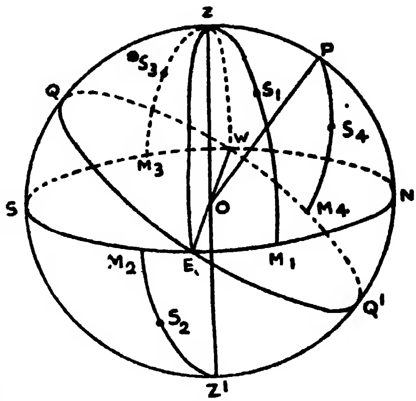

Fig. 2.3a

The position of a point in a plane is defined by its Cartesian co-ordinates with reference to two fixed straight lines as axes. Similarly, the position of a point on a sphere can be defined by suitably chosen *spherical co-ordinates* with reference to a fixed great circle and a fixed point on it as origin. Different systems of co-ordinates in use in Astronomy are given below.

(1) *Altitude and Azimuth:* In this system the horizon is the reference circle. *Altitude* of a celestial body is its distance from the horizon measured along the vertical drawn through it. The range of measures of altitudes is from 0° to 90°. When the body is below the horizon, its altitude may be considered negative. Often, instead of altitude its complement called the Zenith distance (abbreviation used is Z. D.) is employed.

Azimuth is the distance of the foot of the vertical drawn through the body, measured along the horizon, from a point on it, usually the south point. The range of values of azimuth is from 0° to 180°, east and west. The reckoning of azimuth varies in practice; but it should always be given in a way which leaves no room for ambiguity.

In Fig. 2.3a, ZS_1M_1, $Z'S_2M_2$, ZS_3M_3 are verticals drawn through the celestial bodies S_1, S_2, S_3. The altitudes of S_1 and S_3 are S_1M_1 and S_3M_3 respectively. The altitude of S_2 is negative, the magnitude being S_2M_2. Considering M_1S_1 Z S_3M_3 to be the same great circle, the altitude of S_3 may not be indicated by the longer arc S_3 Z S_1M_1 but by the shorter arc S_3M_3. The azimuth of S_1 is SM_1. Supposing it to be equal to 150°, the azimuth should be given as S 150° E, meaning that it is measured from the south along the eastern half of the horizon.

Though this system is the most obvious, it is inconvenient in one important respect. Stars which form the largest group of

celestial bodies and have no relative motion among themselves, do not have permanent co-ordinates in this system; for, their positions constantly change with respect to the horizon.

A system in which the co-ordinates of stars remain constant is the following:

(2) *Right Ascension and Declination:* In this system, the equator is the reference circle and the First point of Aries the origin.

The *Right Ascension,* briefly written R. A., of a celestial body is the arc measured *eastward* on the equator, from the First point of Aries to the foot of the secondary through the celestial body drawn to the equator.

Unless the horizon is indicated on the celestial sphere, the east point and the eastward direction would be meaningless. In the definition above, however, by the eastward direction we mean the direction in which the sun moves on the celestial sphere, relative to stars. The movement of the sun on the celestial sphere, relative to stars, is eastward to any observer on the earth. This is why we use the word 'eastward' in our definition.

The range of measures of R.A. is therefore from 0° to 360°. Usually R.A. is measured in hours instead of degrees, the equivalent of one hour being 15°. This is due to the fact that the R.A. is determined by observing the interval of time between the passages of the First point of Aries and the celestial body over the meridian (SEC. 5.1).

The *Declination,* briefly written Decl., of a celestial body is its distance from the equator measured along the secondary drawn through the body.

The range of measure of Decl., is from 0° to 90°, positive if the body be situated in the northern hemisphere and negative if in the southern hemisphere. Instead of Decl., its complement called the *North Polar distance* is used when convenient.

In Fig. 2.3b, S_1, S_2, S_3 represent stars; PS_1M_1, PS_2M_2, PS_3M_3 are secondaries to the equator drawn through them. The R.A. of S_1 is measured by the arc γM_1 or rather its equivalent in hours. The R.A. of S_3 is not the shorter arc γM_3 but the longer arc $\gamma M_1 \Omega M_3$, the measurement is to be always towards the east beginning from γ.

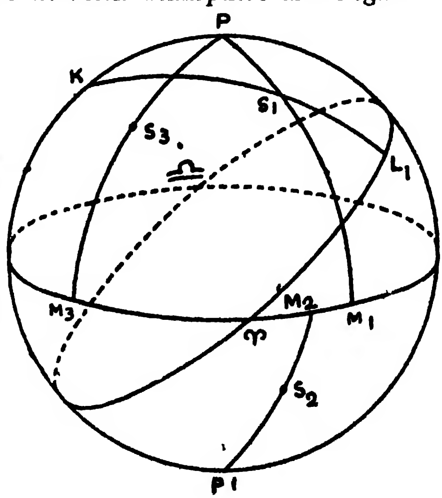

Fig. 2.3b

Decl., of the star S_1 is the arc S_1M_1 and not the supplementary arc. Decl., of S_2 is S_2M_2 and negative. Suppose $S_2M_2 = 30°$; the Decl., of S_2 should be given as $-30°$ or $30°$ south.

The Greek letters α and δ are generally used to indicate R.A. and Decl.

It has been said that the equator is fixed among stars Though this is substantially true over short periods, the equator really has a very slow motion of its own (SEC. 14.4). The ecliptic however has been found to be the most permanent great circle among stars; and we have a third system of co-ordinates with the ecliptic as the reference circle:

(3) *Celestial latitude and longitude: Celestial latitude* of a body is its distance from the ecliptic measured along the secondary drawn through it, to the ecliptic.

The range of measures of celestial latitude is from 0° to 90°. north or south.

Celestial longitude of a body is the arc measured eastward along the ecliptic from the First point of Aries to the foot of the secondary, drawn through the body.

In Fig. 2.3b, the celestial latitude of the star S_1 is the arc S_1L_1 and the celestial longitude is the arc $\gamma\ L_1$, where $K\ S_1L_1$ is the secondary to the ecliptic drawn through S_1.

Sometimes it is convenient to define the position of a celestial body by co-ordinates, one of which is affected by diurnal motion but the other is not. Such a system is the following:

(4) *Declination and Hour angle:* Declination has already been defined. The *Hour angle* of a celestial body is the angle between the meridian and the secondary to the equator drawn through it. It is reckoned from 0° to 360° from the meridian towards the west. It will be noted that the hour angle gives the time which has elapsed since the secondary was last coincident with the meridian *i.e.*, the body was last on the meridian. Like the R.A., the hour angle is also usually measured directly in time.

Definition: The secondary to the equator drawn through a body is often called its *Hour circle*. Since the declination is also measured on it, it is also called the *Declination circle*.

In Fig. 2.3a, the declination of the star S_4 is of course S_4M_4. The hour angle is the equivalent in time of the arc QM_4 on the equator.

2. 4. The following theorem is of importance when the meridian altitude is required but the meridian is not previously determined, as for example, in observation of meridian altitude at sea.

Theorem: The altitude of a star is greatest when it is on the meridian.

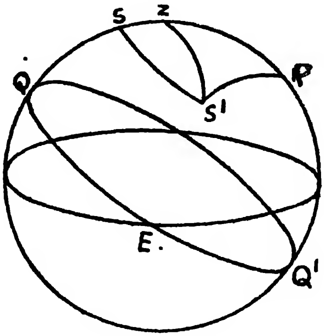

Fig. 2 4a

Let Z be the zenith and P the celestial pole, and S and S′ be two positions of a star, on and out of the meridian. Consider the spherical triangle PZS′. PZ+ZS′ > PS′ > PZ+ZS, since PS = PS′. ∴ ZS′ > ZS. Hence the complement of ZS′ < the complement of ZS *i.e.*, altitude of S > altitude of S′. The meridian altitude is therefore the greatest.

Examples Worked Out

1. Find when the sun's R.A. will be 120°, assuming it to change uniformly.

The sun's R.A. is 0° on March 21.

His R.A. will be 120° after (120°/360°) of 365¼ days *i.e.*, 122 days approximately. Counting the days of the months after the 21st. March we have March 10; April 30; May 31; June 30; the total is 101 days. Therefore the date required is the 21st July.

2. If the R.A. of a star is equal to its latitude, prove that its declination must be equal to its longitude.

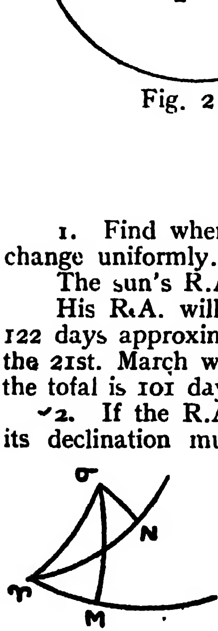

Fig. 2.4b

Let σ be the star and γ the first point of Aries. Draw secondaries σ M and σ N, through the star, to the equator and the ecliptic respectively. Join $\gamma\sigma$. Consider the two spherical triangles $\gamma\sigma$M and $\gamma\sigma$N. $\gamma\sigma$ is common; $\sigma N = \gamma M$. Therefore the right angled triangles are congruent. Hence $\gamma N = \sigma M$.

3. Show that the angle between the declination circle and horizon is least when the declination circle passes through the east point.

The declination circle being a secondary to the equator, the locus of one of its poles (Q, say) is the equator itself. The zenith Z is again a pole of the horizon. Hence ZQ is the angle between the declination circle and horizon. Obviously ZQ is least when Q is on the meridian as shown in Fig. 2.4a. But the declination circle having its pole Q on the meridian passes through E; *i.e.*, the angle between the declination circle and horizon is least when the declination circle passes through the east point.

4 A star is on the meridian 15° above the pole at midnight to-night. Where will it be at midnight (i) six months hence; (ii) a year hence, supposing the sun's apparent motion in the ecliptic to be uniform?

On the day of observation the hour angle of the star is less than that of the sun by 180°. Now we know that the hour angle of a star increases by 360° over that of the sun in course of a year. Hence in 6 months the difference of their hour angle will be zero. So the star will be in its lower transit at midnight *i.e.*, it will be 15° below the pole at midnight six months hence. In one year the difference of their hour angle will again be 180°. So the star will be again 15° above the pole at midnight a year hence.

Exercise 2

1 How can a planet be distinguished from a star? (C. U. 1914)

2 Describe the apparent daily motion of stars.

3 Define Zenith, Nadir, Celestial Pole, Horizon, Ecliptic, Prime vertical, Hour angle, Hour circle, North polar distance, the Zodiac.

4 Define the three systems of co-ordinates (1) Altitude and Azimuth (2) R. A. and Decl. (3) Latitude and Longitude.

5 The pole of a great circle is given by R. A. = 190°, Decl = 28°. Find the R. A. of its points of intersection with the equator.

6 Give the R. A. and Decl., and Latitude and Longitude, of the sun on the 21st March, the 21st June, the 23rd September, and the 21st December.

7 Show that a star's altitude is the least angle between the star and any point on the horizon.

8 The R. A. of an equatorial star is 90°. Find the time when it rises on the 21st March. When does it cross the meridian on the same date?

9 The R. A. of a star is 270°, when does it cross the meridian on the 18th June and the 24th June? (Assume that the rate of increase of R. A. of the sun during the period = the average rate over a year.)

10 The sun crosses the meridian of an observer on a particular day at a distance of 75° from the pole and 30° from the zenith. What will be the meridian zenith distance of a star diametrically opposite the sun on the day?

11. If the longitude of a star be equal to its declination, prove that its latitude is equal to its right ascension. (C.U. 1909).

3. THE EARTH: ITS FIGURE AND ROTATION

3. 1. It will be seen in subsequent chapters that the shape, size and motion of the earth have important bearing on the study of Astronomy, because we have to make all observation from the surface of the earth. We therefore first discuss these problems.

It is obvious that the earth's surface is neither plane nor concave. For, in either case we could have seen through a telescope a fairly distant object on the surface of the earth; for example, we could have seen a distant light-house or a distant island out at sea from the sea shore. As the example in SEC. 1.3 shows, the surface is really convex. Further, that it is spherical appears from the following consideration. From our complete knowledge of the motion of the sun, it is possible to tell his direction at any time. At a lunar eclipse he is found to be exactly opposite the moon; and the observed shadow on the moon must be of the earth. Since it is circular every time, we conclude that the earth is spherical.

Beyond such general consideration, accurate investigation of both the shape and the size of the earth can be carried out in the following way.

3. 2. Let A and B be two points on the earth's surface, several miles apart, and B due north of A (Fig. 3.2a). The celestial meridian at the two points will be the same. Let AH and BK be the tangents to the arc AB of the earth's surface at A and B. Let AS and BS′ be the direction of the same star when on the meridian. As the star is infinitely distant, AS is parallel to BS′. Draw normals to the earth's surface at A and B to meet at O. Produce AH to meet BS′ at G and BK at H.

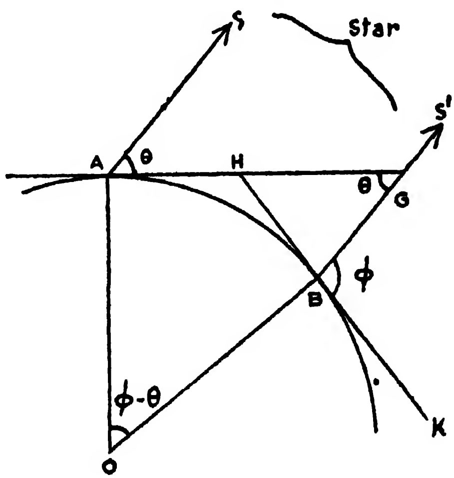

Fig. 3.2a

From the triangle BGH, $\angle$ GHB $= \angle$ GBK $- \angle$ HGB $= \angle$ GBK $- \angle$ SAH, since AS is parallel to BS′ and AG meets them; $\angle$ GHB $= \phi - \theta$, where ϕ and θ are the meridian altitudes of

the star at B and A respectively. From the quadrilateral OAHB, since the angles at A and B are right angles, $\angle$ AOB $= \phi - \theta$. Assuming that the arc AB is sufficiently small compared with the whole circumference, AO = BO = the radius of curvature of the arc AB, σ, say. Hence $\sigma(\phi - \theta)$ = arc AB, $\phi - \theta$ being in radians.

$$\text{Therefore, } \sigma = \frac{\text{arc AB}}{\phi - \theta} \qquad \ldots\ldots \qquad \ldots\ldots \qquad (3.2)$$

Now θ and ϕ can be observed and the arc AB accurately measured by the method of *Triangulation*. Hence σ follows from equation (3.2). It is found that σ has approximately the same value at different points of not only the same but also different meridians. The earth is therefore nearly spherical. The equatorial radius has been estimated to be roughly 4000 miles; the polar radius is a few miles smaller. (See Appendix B for more accurate values of the radii).

Triangulation: Direct measurement of a long distance is not possible with any high accuracy; for there may be irregular country between the extremities and the errors of the measuring implement will be multiplied as many times as it is used. In triangulation, a short horizontal length AC is measured with every possible care (Fig. 3.2b). Suppose we want the distance AB along the meridian of A. Let G be a point in the same latitude as B. Join A and G by a series of triangles ACD, CDE, DEF and EFG. All the angles of the triangles are carefully measured. They therefore can be solved successively and the lengths AD, DF and FG are obtained. The length AB can then be calculated as the sum of the projections of AD, DF and FG.

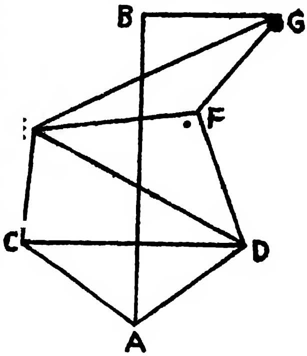

Fig. 3.2b

3.3. Next let us consider the question of motion of the earth. The very fact that such a large number of celestial bodies as stars, possess one common motion will incline us to believe that the motion is only relative and that the earth is rotating in the opposite direction *i.e.*, from west to east. This conjecture is strengthened by dynamical consideration. Enormous force ($-\omega^2$. r $\times$ mass, where ω is the angular velocity and r is the radius of the orbit) is required to keep a star in such a huge circular path with its fairly large angular velocity. It is unbelievable that the earth can exert such a huge force at such an enormous distance.

Besides conjecture, the following experiments provide evidence of the earth's rotation.

(a) *Newton* (English Mathematician: 1642-1727) *proposed the following experiment to find out if the earth rotates.*

Suppose AB is a vertical tower at a point A on the terrestrial equator and O is the earth's centre; OAB is then a straight line (Fig. 3.3a). If the earth rotates about an axis perpendicular to the plane of the paper through O, in the direction of the arrowhead in the figure, B the top of the tower will have a larger velocity V and A the bottom a smaller velocity v. For, during a complete rotation of the earth, B describes a larger circle and A a smaller, in the same time. Hence if a heavy ball be dropped from B, it should reach the bottom a little ahead *i.e.*, a little to the east of A. The deviation of a particle dropped from a tower of any height, situated at any latitude, can be mathematically worked out on the hypothesis of the earth's rotation. Experiments have been performed but are not quite convincing, on account of the smallness of the deviation to be expected which might as well be due to minor accidental causes.

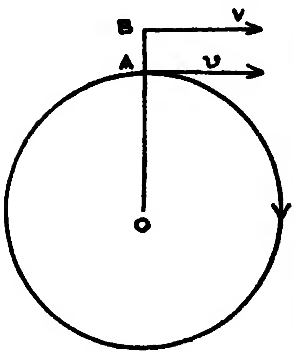

Fig 3.3a

(b) An entirely convincing proof is provided by the elegant *pendulum experiment of Foucault* (French Physicist: 1819-68).

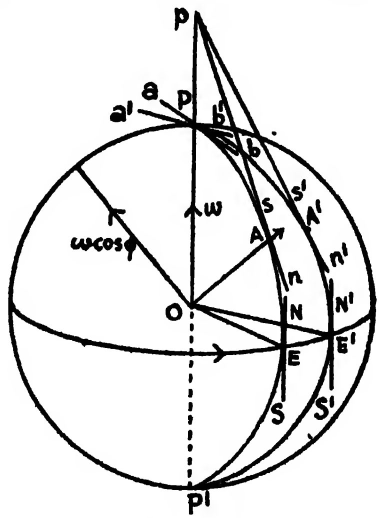

Fig. 3.3b

The underlying principle of the experiment is easily understood. In Fig. 3.3b, the sphere with centre O, represents the earth. Assuming that it rotates about the axis POP′ in the direction shown by the arrowhead, the meridian P′ EAP will, in an interval, move to the position P′E′A′P. The tangents to the meridian P′EAP:—(1) NS at E on the terrestrial equator, (2) ns at A at an intermediate latitude and (3) ab at P the north pole move to the positions N′ S′, n′ s′, and a′ b′ respectively. Now NS and N′S′ are both parallel to the diameter P′OP and are therefore parallel to one another: *i.e.*, the direction of the north and south line at E does not change on account of the rotation, sn and s′ n′ produced meet the axis POP′ at the same point p; for OA = OA′ and ∠AOP =

$\angle$ A′ OP being complements of the same latitude. Hence the hypotenuses must be of the same length Op. It follows that ns changes direction with the rotation of the earth. That ab changes direction with the rotation is at once obvious.

Suppose now that freely suspended pendulums are set swinging in the vertical planes through NS, ns and ab at E,A, and P respectively. The motion of the earth carries the pendulums forward but cannot cause their planes to rotate about the verticals through their points of suspension. The lines ns and ab will therefore gradually separate from the planes of the respective pendulums; but the line NS will not.

The experiment was first performed by Foucault himself in the Pantheon in Paris. The bob of the pendulum was suspended by a fine wire more than 200 ft. long. The pendulum therefore moved very slowly and encountered very little resistance of air and so continued to swing for a long time. The pendulum was started in a true plane by burning off a thread which held the bob aside. A ridge of sand was placed round the pendulum on the floor; and a pointer on the bob made marks on it at every swing. The successive marks did not coincide, showing that the plane of the pendulum deviated from the line in which it was originally set swinging. It was thus demonstrated that the earth rotates about its axis.

The rate of deviation of the plane of a pendulum at any latitude can be calculated.

Let the latitude of the place A $= \angle$ EOA $= \varphi$ and let $\omega =$ the rate of rotation of the earth. Angular rotation being a vector quantity may be represented by a straight line normal to the plane of rotation, the sense of the straight line being related to the direction of rotation by the right hand screw rule. Accordingly, ω may be represented by a length along OP, the direction of rotation being as shown by the arrow-head in the figure. ω can be resolved by the parallelogram law, its component along OA $= \omega \cos$ POA $= \omega \sin \varphi$. The perpendicular component does not cause any relative motion of the plane of the pendulum, just as ω itself does not cause any at E. The effect of the component along OA is to make the line ns rotate, the north end towards the west, with the angular velocity $\omega \sin \phi$.

Hence the plane of the pendulum will appear to rotate in the opposite direction with the same angular velocity $\omega \sin \varphi$. The period of a complete rotation is therefore $\frac{2\pi}{\omega}$ cosec φ = (23 hr. 56 min. 4 sec) cosec ϕ.

Ocean currents and trade winds undergo westward deviation when directed *towards* the equator, and eastward deviation when directed *away from* the equator. These facts are in agreement with the theory of rotation of the earth about an axis.

Rotation of the earth completely explains the apparent daily

motion of stars and for the present we consider them to have no residual motion to be further explained.

Examples Worked Out

1. Taking the earth to be a sphere find the distance passed over by an observer from A to B on the same meridian if the zenith distance of a star changes by 10° at the end of the journey.

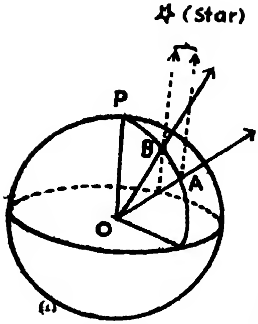

Fig. 3.3c

Let O be the centre of the earth The $\angle$ AOB = 10°. Now 10° = $\pi/18$ radians = the arc AB/4000, where arc AB is in miles. $\therefore$ Arc AB = $(4000\pi) \div 18 =$ 698.4 miles, approximately.

2. Supposing the earth to rotate with the same angular velocity as at present, but in the opposite direction, what would be the length of a civil day and the number of civil days in a year? (C.U. '41).

As the earth is supposed to rotate with the same angular velocity as at present, the number of revolutions of a star with respect to an observer will be $366\frac{1}{4}$ in course of a year. Again the number of revolutions of the sun with respect to stars is 1 in course of a year. But the sun moves in the same direction relative to stars in this case. Therefore the number of revolutions of the sun with respect to the observer will be $366\frac{1}{4}+1=367\frac{1}{4}$ in course of a year. In other words, the number of civil days in a year $=367\frac{1}{4}$. But the length of a year is equal to $365\frac{1}{4}\times 24$ mean solar hours. Therefore the length of the civil day of our problem is equal to $(365\frac{1}{4}/367\frac{1}{4})\times 24$ mean solar hours = 23 hr. 52 min. approximately.

3. Two ports are at the same latitude but their longitudes differ by 180°. Prove that the distance between them measured along the parallel of latitude differs from that measured along the great circle joining them by $a\ (\pi\cos\phi - \pi + 2\ \phi)$ where ϕ is the latitude of the ports and a is the radius of the earth For what latitude the difference will be maximum?

Since the longitudes of the ports differ by 180°, the great circle joining them will pass through the poles of the equator.

Distance measured along the parallel of latitude $= \pi a'$ where a' is the radius of the parallel of latitude $= \pi a\cos\varphi$

Distance measured along the great circle joining the ports

$$=2\left(\frac{\pi}{2}-\phi\right)a=(\pi-2\varphi)a.$$

$\therefore$ Distance along the parallel of latitude – Distance along the great circle $=\pi a\cos\varphi-(\pi-2\varphi)a=a(\pi\cos\varphi-\pi+2\varphi)$.

For maximum $\frac{d}{d\varphi}[a(\pi \cos \varphi - \pi + 2\varphi)] = 0$

i.e. $-a \sin\varphi . \pi + 2a = 0$. *i.e* $\sin \varphi = \frac{2}{\pi}$.

Hence the difference will be maximum when $\varphi = \sin^{-1}\frac{2}{\pi}$

4. A person starting at sun-rise to travel round the earth's equator finds the sun to cross his meridian t times and completes his journey at sun-set Prove that the speed of the traveller = $\frac{\pi r}{12t - 18}$ or $\frac{\pi r}{12t + 6}$ units of distance per hour according as he travels east or west, where r is the radius of the earth.

Let ω_1 and ω_2 be the angular velocities of the traveller and the sun about the centre of the earth. Consider the case when the man travels eastward. Angular velocity of the sun relative to the man $= \omega_1 + \omega_2$. Now during the whole journey the angle described by the sun relative to the traveller $= \frac{\pi}{2} + (t-1)2\pi + \frac{\pi}{2}$

(the angle described by the sun from morning to next noon $= \frac{\pi}{2}$

As there are $(t-1)$ other transits the angle described by the sun on that count $= (t-1)2\pi$. Besides the angle described from the last noon to sun-set $= \frac{\pi}{2}$)

$$\therefore \text{ the time taken } = \frac{\frac{\pi}{2} + (t-1)\,2\pi + \frac{\pi}{2}}{\omega_1 + \omega_2} = \frac{(t-1)2\pi + \pi}{\omega_1 + \omega_2}$$

But this is the time in which the man completes a revolution on the surface of the earth Hence the time is also $= \frac{2\pi}{\omega_1}$. Equating the two we have

$$\frac{\pi + (t-1)2\pi}{\omega_1 + \omega_2} = \frac{2\pi}{\omega_1} \quad \text{whence} \quad \omega_1 = \frac{2\omega_2}{2t-3} = \frac{2.2\pi}{(2t-3)24} \text{ radians/hour}$$

$$\therefore \text{ the required speed } = \frac{\pi r}{(2t-3)6} \text{ units of distance/hour}$$

$$= \frac{\pi r}{12t - 18} \text{ units of distance/hour.}$$

The other case may similarly be worked out and is left as an exercise.

Exercise 3

1. Give a proof of the earth's rotation.
2. How should a comparatively long distance over a rough country be measured?
3. How is a degree of meridian measured? Assuming the length of a degree to be $69\frac{1}{2}$ miles, find the earth's radius in miles.
4. At a place due north of another, the elevation of the same star, when on the same meridian, is $1^\circ\ 30'$ greater. What is the distance between them? (Consider the earth to be a sphere of radius 4000 miles.)
5. A ball is dropped from a tower 100 ft. high and situated on the equator. Calculate how far away from the bottom of the tower the ball will touch the ground. (Take $g=32$ ft./sec^2 the radius of the earth $=4000$ miles).
6. A Foucault's pendulum is set vibrating at a place. After one sideral day *i.e.*, 23 hr. 56 min. 4 sec., it is vibrating in the same plane for the second time. Find the latitude of the place.
7. A, B, C are three vertical posts of equal height fixed two miles apart along a straight line in a canal. Find how much below B will AC pass. (Take the radius of the earth to be 4000 miles).
8. How far should a man travel northward from the equator in order that the altitude of the pole might become 10°? Assume the radius of the earth to the 4000 miles. (C.U. 1940).
9. If a person travelling eastward go round the world, he will at the end of his journey appear to have gained a day. On the other hand, if he travel westward, he will appear to lose a day. Explain. (C.U. '26, '28, '33, '36, '42).
10. The height of a light-house above the sea level is known to be b. Show that its distance from a ship when its top is just visible to an observer on the ship at a height a above the sea level is $r\left(\sqrt{\frac{2a}{r}}+\sqrt{\frac{2b}{r}}\right)$ neglecting $\frac{a}{r}, \frac{b}{r}$ and their higher powers.

4. THE EARTH: DIURNAL PHENOMENA

4. 1. Diurnal phenomenon at any place depends on the position of the celestial pole with respect to the horizon of the place. The position is known as follows:

Theorem The altitude of the celestial pole at any place is equal to the latitude (terrestrial) of the place

Let the circle EA (Fig. 4.1) on the sphere, centre O, represent a meridian of the earth, E, the point on the equator and A, the point of latitude φ on the meridian so that $\angle$ EOA $= \varphi$. The celestial pole, situated on the celestial sphere, is infinitely distant Hence its directions OP and AP from O and A respectively, are parallel. But OP is perpendicular to OE $\angle$ PAH is the altitude of the celestial pole, for the tangent AH at A is the horizon of the place. Now $\angle$ PAH is the complement of $\angle$ ZAP. Again, $\angle$ EOA which measures the latitude of A is the complement of $\angle$AOP. But $\angle$ AOP $=$ $\angle$ ZAP, since OP is parallel to AP. Hence $\angle$ PAH $= \angle$ EOA $= \varphi$, and the theorem is proved.

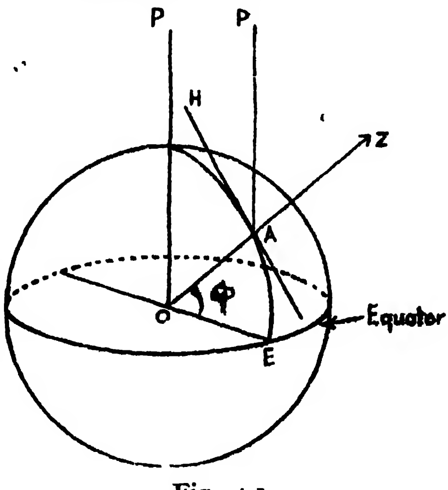

Fig. 4.1

Two different representations of the celestial sphere are given in Fig. 4.2a and Fig. 4.2b. In the first, the horizon is shown; and the position of the celestial pole (and therefore of the equator) is determined by the theorem just given. All celestial bodies in Fig. 4.2a are in diurnal motion from east to west in small circles round the celestial pole. On the other hand, stars occupy fixed positions in Fig. 4.2b. The diurnal path of a celestial body can be constructed in this figure by imagining the body to be revolved round the celestial pole thus tracing out a small circle. The corresponding small circle in Fig. 4.2a can be easily located. The body rises or sets according as it is at one or the other point of intersection of the small circle with the horizon. To explain the general diurnal phenomenon at any place, both the representations may be used side by side.

4. 2. It has already been stated (SEC. 2.2) that the sun is at the First Point of Aries and the First Point of Libra on the 21st. March and the 23rd. September respectively. These two days are called the *Equinoxes*—the first, the *Vernal equinox* because it marks the beginning of Spring; and the second, the *Autumnal equinox* because it marks the beginning of Autumn.

The sun is 90° from the First Point of Aries in the forward direction (in the direction in which the sun moves relative to stars) on the 21st. June and 90° forward from the First Point of Libra on the 21st. December. These two days are called the *solstices*.

Equinox means that the duration of day is equal to the duration of night. At both the vernal and the autumnal equinox, day and night are of equal duration all over the earth—as will be presently explained. The word *solstice* literally means 'standing still'. At solstices, the sun seems to have stopped in his march away from the equator.

Definition: The secondary to the equator through the solstitial points is called the *Solstitial colure*.

Definition: The secondary to the equator through the equinoctial points is called the *Equinoctial colure*.

Definition: The small circle parallel to the equator through the summer solstitial point is called the *Tropic of Cancer*.

Definition: The small circle parallel to the equator through the winter solstitial point is called the *Tropic of Capricorn*.

The word Tropic means 'turning point'. The sun turns back south from the small circle called the Tropic of Cancer. It is so called because the sun used to be at the fourth sign of the Zodiac namely Cancer, at the summer solstice. Similarly, the sun turns back north from the Tropic of Capricorn, which is so called because the sun used to be situated at the tenth sign of the Zodiac namely Capricorn, at the winter solstice. At present, the sun does not occupy a position in the constellations Cancer and Capricorn during the solstices.

We explain below the phenomena of equinoxes and solstices. Let Fig. 4.2a be the celestial sphere of a particular place; the horizon is represented by a fixed great circle, and all celestial bodies are in diurnal motion on the sphere. Let Fig. 4.2b be another representation of the celestial sphere on which stars have fixed positions, and therefore the horizon of any place should be moving and is not represented at all. The sphere is thus independent of the position of the observer. In both the figures, P is the celestial pole, EQ the equator, LK the tropic of Cancer, L'K' the tropic of Capricorn, CD any parallel to the equator. Besides Z is the zenith and NES the horizon in Fig. 4.2a.

At the equinoxes, the sun is at γ or Ω *i.e.*, on the equator (Fig. 4.2b). His diurnal path is obtained by rotating him round P, the path is therefore the equator itself. The equator and the horizon are both great circles, and bisect one another (Fig. 4.2a). So half the diurnal path is above and half below the horizon *i.e.*,

duration of day is equal to duration of night and this is so at any place of the earth.

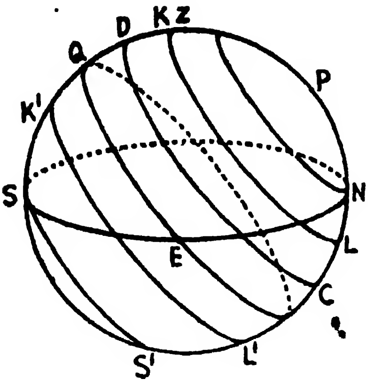

Fig. 4.2a

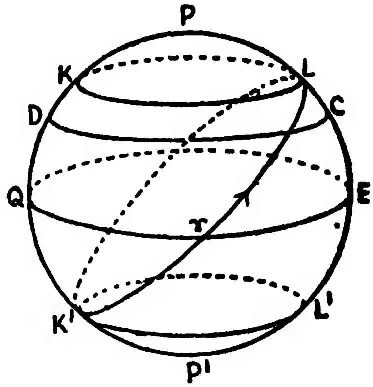

Fig. 4.2b

At the summer solstice, the sun is at L (Fig. 4 2b) 90° in front of γ. The tangent to the ecliptic at L is parallel to the equator. Hence for a few successive days the sun keeps practically the same distance from the equator. His diurnal path on these days is practically the same small circle LK (Figs. 4.2a & b). So he seems to 'stand still' in his march north of the equator. This is also the circle from which he turns back south. The *circle* is called the Tropic of Cancer and the *day*, a solstice. A similar explantation holds with regard to the winter solstice and the Tropic of Capricorn.

4. 3. Below we explain in detail the general diurnal phenomenon in three representative places: (1) at the equator (terrestrial), (2) at an intermediate latitude and (3) at the north pole.

(1) Fig. 4.3a represents the celestial sphere at the equator of the earth. The latitude of the place is zero and the celestial pole is on the horizon and coincides with the north point, N. The celestial equator is perpendicular to the celestial horizon and passes through the zenith. The path of any star is a small circle such as AC round P and is bisected by the horizon. Hence all stars are visible, half the time above and half the time below the horizon.

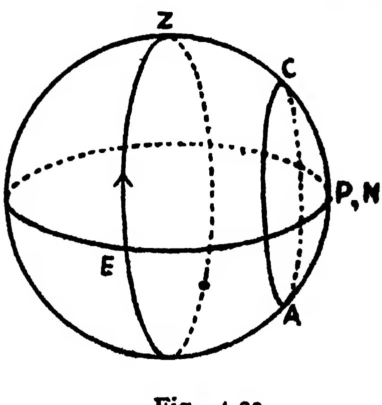

Fig. 4.3a

The diurnal path of the sun any day is similarly a small circle round P and is bisected by the horizon. So duration of day is equal to duration of night throughout the year.

(2) Fig. 4.2a represents the celestial sphere at an intermediate north latitude. The altitude of the pole P = PN = the given latitude.

The southern stars within the zone bounded by the small circle SS′ round P through the south point S will have their diurnal paths completely below the horizon and will be never visible. On the other hand the northern stars within the zone bounded by the small circle NN′ round P through north point N, will have their diurnal paths completely above the horizon and would be constantly visible except for the glare of the sun. Such stars are called *Circumpolar stars.*

Stars in the belt bounded by NN′ and SS′ will have parts of their diurnal paths above and parts below the horizon. These stars therefore rise and set.

It has already been explained that at the equinoxes duration of day is equal to duration of night at all places and therefore at the place in question. From the 21st. March to the 23rd. September, the sun is north of the equator (Fig. 4.2b); his diurnal path on any day during the period is a small circle such as CD (Fig. 4.2a) north of the equator EQ. More than half of such a circle is above the horizon; consequently duration of day is longer than duration of night. The reverse is the case when the sun is south of the equator *i.e.,* during the period from the 23rd. September to the 21st. March.

If a small circle representing the diurnal path of the sun happens to be completely above the horizon, the sun will not set during 24 hours; the phenomenon is known as the ***Midnight sun*** and occurs in high latitudes.

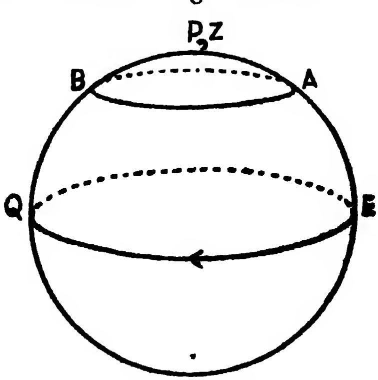

Fig. 4.3b

(3) Fig. 4.3b represents the celestial sphere at the north pole. The latitude being 90°, the celestial pole P coincides with the zenith; the equator EQ coincides with the horizon.

The path of any star is a small circle such as AB round P parallel to the horizon EQ. All stars are circumpolar. Half of them are always above the horizon and so always visible. Those below the horizon are never visible.

From the 21st. March to the 23rd. September the sun is north of the equator (Fig. 4.2b) and therefore continuously above the horizon (Fig. 4.3b). It is therefore day during the whole

period of six months. Similarly, the sun remains continuously below the horizon and so it is night for the whole period from the 23rd. September to the 21st. March.

4.4. *Changes of seasons* are principally due to variation of the amount of the sun's heat received at a place on the earth's surface. Consider a place in the northern hemisphere of the earth. Astronomically the year is divided at such a place into four seasons as follows:

Spring:	From the 21st. March to the 21st. June.
Summer:	From the 21st. June to the 23rd. September.
Autumn:	From the 23rd. September to the 21st. December.
Winter:	From the 21st. December to the 21st. March.

Let Fig. 4.2a represent the celestial sphere of the place. During the period, 21st. March to 23rd. September, the sun is north of the equator, northernmost at L on the 21st. June (Fig. 4.2b). His diurnal paths during this period are small circles like CD, all lying parallel to and north of the equator; the northernmost is KL. Obviously greater parts of these circles lie above and smaller parts below, the horizon. And the more northerly the circle is, the larger is the ratio of the portion above to the portion below. It follows that days are longer than nights, the day being the longest on the 21st. June. This is one cause of accumulation of heat at the place during the period.

Another and a very powerful cause of accumulation, is the fact that the sun reaches greater altitudes during the period, in comparison with the remainder of the year. This will be obvious when we compare altitudes of points, say, on the small circle KL, with corresponding points (*i.e.*, points on the same secondary to the equator) on the small circle K′L′ which is equally south of the equator.

Now, in Fig. 4.4, { XA, XC } and { ZA, ZB } represent two beams of sun-rays of equal breadth, the first from the sun at zenith and the second from the sun at a smaller altitude. The same quantity of heat energy is spread over unequal areas, smaller (AB) in the first and larger (AC) in the second case. Heat received per unit area is therefore greater in the first case.

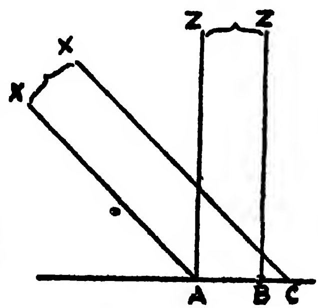

Fig. 4.4

Owing to the combination of the two causes (1) longer days and (2) greater altitudes of the sun, heat received during the period is greater. Moreover night being shorter, loss of heat by radiation is smaller. The con-

tribution to the accumulation of heat, of the days in the middle of the period, is of course greater than that of the days either at the beginning or at the end of the period. But the *total* accumulation of heat as distinct from the contribution of individual days is the greatest during the latter half of the period *i.e.*, from the 21st. June to the 23rd. September. So this period is called *Summer*.

For opposite reasons, the period from the 21st. December to the 21st. March is the coldest and called *Winter*. Intermediate periods of moderate temperature preceding Summer and Winter are called *Spring* and *Autumn* respectively.

Local and meteorological factors modify temperature in any season but the broad changes are directly governed by the sun.

It should be stated that in southern latitudes Winter corresponds in time to Summer of northern latitudes and vice versa.

4. 5. *Twilight* is the period of partial light immediately before sun-rise or immediately after sun-set. Duration of twilight is supposed to be limited by the appearance of stars of the sixth magnitude (which by the way are the faintest visible to the naked eye) near the zenith. It has been ascertained that they become visible when the sun is about 18° below the horizon. We shall adopt this criterion for the calculation of duration of twilight.

The phenomenon is due to the presence of dust particles hanging in the atmosphere. When the sun is below the horizon, his rays still illumine these particles in the upper regions of the atmosphere which in turn scatter light to the surface of the earth causing partial light.

To calculate the duration of twilight at the equator at an equinox. See - 456

Fig. 4.3a represents the celestial sphere at the equator; the celestial pole P coincides with the north point N. The sun is situated on the equator (it being equinox) and his diurnal path for the day is the equator itself. On the celestial sphere of the place the equator coincides with the prime vertical, as can be seen from the figure. Consider twilight before sun-rise; it begins when the sun is at S, 18° below the horizon.

Duration of twilight = the time taken by the sun to describe the arc ES (E is the east point)

$$= \frac{18}{360} \times 24 \text{ hr.} = 1 \text{ hr. } 12 \text{ min.}$$

Duration of twilight at any place and at any time can be calculated by solving spherical triangles. The actual calculation is beyond our scope; but the principle is easily understood.

Let Fig. 4.5a represent the celestial sphere for the particular place. The arc PN which is the altitude of the celestial pole is known, being equal to the given latitude of the place. Let it be φ.

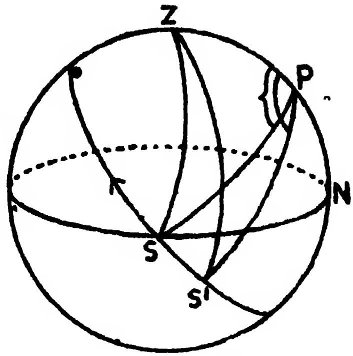

Fig. 4.5a

Let SS′ denote the diurnal path of the sun on the given day. Let twilight begin when the sun is at S′, so that the arc ZS′ is 108°. It ends when the sun is at S on the horizon. Duration of twilight is proportional to the angle SPS′. Now the spherical triangles ZPS and ZPS′ can be solved. For, $ZP = 90° - \varphi$, $ZS = 90°$, $ZS' = 108°$, $PS = PS' = 90° - \delta$, the polar distance of the sun on the particular date. So all the sides of both the triangles are known; and angles ZPS and ZPS′ can be calculated. By taking their difference, the angle SPS′ and hence the duration is obtained. It therefore follows that duration of twilight depends on two quantities (1) *the latitude φ of the place, and* (2) *the declination δ of the sun on the day.*

To find the condition that twilight may last all night at a place.

Let φ = the latitude of the place and δ = the declination of the sun on the day.

Let Fig. 4.5b represent the celestial sphere of the observer and as usual let Z, P, N, E, EQ represent the zenith, pole, north and east points, and the equator respectively. Let ADBC represent the diurnal path of the sun; arc NP= φ and arc AQ= δ. The zenith distance of the sun is greatest at his lower transit *i.e.,* when he is at A. For take any other point D on the path. $ZD < ZP + PD$ *i.e.,* $< ZP + PA$ (for PA=PD=the north polar distance of the sun) *i.e.,* $< ZA$.

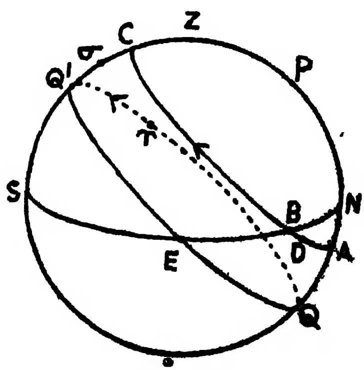

Fig. 4.5b.

Hence twilight will last all night if NA be equal to or less than 18°. Now $PN + NA + AQ = PQ = 90°$ *i.e.,* $\varphi + (18° \text{ or less}) + \delta = 90°$ *i.e.,* $\varphi + \delta = 90° - (18° \text{ or less}) =$ or $> 72°$, which is the condition required.

Examples Worked Out

Find the lowest latitude at which it is possible to have a midnight sun. (C.U. '24, '45).

Let Z, P, N be the zenith, the celestial pole, and the north point. Let S be the sun at his lower transit. His distance from the zenith is now the greatest; and there will be a midnight sun if he is above the horizon in this position.

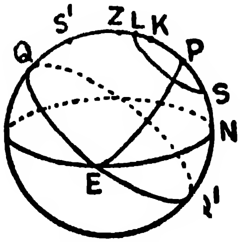

Fig. 4.5c

ZPN is the meridian. PN = altitude of the pole = the latitude of the place $= l =$ or $>$ PS *i.e.*, $90° - \delta$. Therefore l is minimum when $= 90° - 23°\ 28' = 66°\ 32'$.

2. Find the highest latitude at which it is possible to have the sun in the zenith (C.U. '24, '30, '44).

Since the sun is at the zenith of the place, with the notation of the previous example, we must have PS = ZP; or $90° - \delta = 90° - l$, where δ and l (= PN) are the declination of the sun and the latitude of the place. $\therefore$ $l = \delta$. Therefore the largest value of l is $23°\ 28'$.

What is the hour angle of the sun at sun-rise on the 21st. March? (C.U. '40, '44, '46).

The sun is on the equator on the date. His diurnal path is therefore the equator itself. He rises at the point of intersection of the equator and the horizon, *i.e.*, at the east point. The east point being the pole of the meridian, it is easily seen that its hour angle = QQ'E (Fig. 4.5c); *i.e.*, hour angle of the sun at sun-rise is 270°.

4. At a place of north latitude 45°, the Z.D. of a circumpolar star is 15° at upper transit. What is the star's declination?

Let Z and P be the zenith and the pole; and L the star at its upper transit (Fig. 4.5c). The declination $= 90° - PL = 90° - (PZ - ZL) = 90° - (45° - 15°)$; for $ZL = 15°$ and $PZ = 90°$ - lat. $= 90° - 45° = 45°$; $\therefore$ the decl. $= 60°$.

5. Find the inclination of the ecliptic to the horizon when the first point of Aries is rising on the eastern horizon of an observer of latitude 25° N. (C.U. 1917).

The first point of Aries being on the equator rises at the east point. The east point is the pole of the meridian, and lies on the horizon, the equator, and the ecliptic at the instant. Therefore the zenith Z, the celestial pole P, and the pole of the ecliptic K, all lie on the meridian. Moreover it will be seen that the ecliptic lies between the equator and the horizon; so K lies between Z and P. The angle between the ecliptic and the horizon is the angle between their poles, *i.e.*, $= ZK = ZP - KP = (90° - 25°) - 23°\ 28' = 41°\ 32'$.

6. On the longest day at noon, a vertical rod casts a shadow equal to its length on the horizontal ground. Find the latitude of the place.

Obviously, the meridian Z. D. of the sun is 45°. Let Z, P and S′ be the zenith, pole and the sun (Fig. 4.5c). $PZ+ZS'+$decl. of the sun$=90^\circ$. Or $(90^\circ-\text{lat.}) + 45^\circ + 23^\circ\ 28' = 90^\circ$.
$\therefore$ lat.$=45^\circ+23^\circ\ 28'=68^\circ\ 28'$.

Exercise 4

1. The meridian zenith distance of the sun is observed to be 15°, on the 21st. June. What is the latitude of the place? What will be the meridian zenith distance of the sun on the 21st March, at the place?
2. What are the greatest and the least angle made by the ecliptic with the horizon at a place of latitude 30°?
3. Can twilight last all night at a place of latitude (i) 40°, (ii) 60°?
4. Show that at a place of latitude 45°, the interval between the time that a star is on the eastern prime vertical and the time of its setting is exactly half the period of its complete revolution.
5. At a place of latitude 45°, a star rises at the north-east point. What will be its latitude when on the prime vertical?
6. Find the lowest latitude at which it is possible for twilight to last all night. (C.U. several times, Andhra '40).
7. Find roughly for how many days in the year the phenomenon of midnight sun can be witnessed in a place of latitude 75°N. (Nagpur '40).
8. Under what conditions would the azimuth of a star remain constant from rising to transit? (Agra '42)
9. What is the latitude of the place at which it is possible for the ecliptic to coincide with the prime vertical?
10. The decl. of Sirius is $16^\circ\ 36'\ 22''$. Find its meridian altitude at Stockholm and Cape of Good Hope if the latitudes of the places be $59^\circ\ 20'\ 33''$ N and $33^\circ\ 56'\ 23''$ S respectively. (C.U. 1911).
11. How far is the sun from the zenith, at noon on March 21, at a place of latitude $59^\circ\ 16'$ N? (C.U. 1912).
12. Show that at a place on the arctic circle, the azimuth of the sun at rising is greater than the sun's longitude by 90° (C.U. 1912).
13. What are the R.A. and decl. of (i) the pole of the ecliptic, (ii) the sun when his longitude is 6^hhr. (C.U. 1913 & 1914).
14. What is the meridian altitude of the sun at Calcutta during summer solstice? The latitude of Calcutta is $22^\circ\ 54'$ (C.U. 1921).
15. Two stars culminate at the same time and the angular distance between them is 10°. If the declination of one is double that of the other, find the declinations. (C.U. 1932).
16. Show how by solving a spherical triangle, the time of sun-rise or sun-set can be calculated at any place on any day. (C.U. 1940).

17 If a certain star cross the meridian at 11 p m to night, at what time will it cross the meridian (i) to morrow night, (ii) 15 days hence, assuming that the change in the sun's R A is uniform? (C U 1943)

18 Find the length of the day on June 21 at a place of latitude 66° 32′

19 If c_1, c_2, c_3 be the shortest lengths of shadows of equal vertical rods posted at three places on the same meridian, prove that the latitudes $\varphi_1 \varphi_2, \varphi_3$ satisfy the equation

$$\frac{c_1(c_2-c_3)^2}{\tan(\varphi_2-\varphi_3)}+\frac{c_2(c_3-c_1)^2}{\tan(\varphi_3-\varphi_1)}+\frac{c_3(c_1-c_2)^2}{\tan(\varphi_1-\varphi_2)}=0$$

20 Find the declination of a star if it passes through the zenith of an observer in the course of its diurnal motion (Annamalai 1938)

5 ASTRONOMICAL INSTRUMENTS

5. 1. Within the limited scope of our study, we require instruments for two main purposes: (1) accurate measurement of R A. and Decl. of celestial bodies, and (2) accurate measurement of the angular distance between two close points on the celestial sphere

In order to measure R A , the position of the first point of Aries on the equator should be known, say, with reference to the meridian of the observatory. This can be achieved by means of a clock, set at O hr. O min. O sec., when the first point of Aries is on the meridian and regulated to show 24 hr , for a complete revolution of the point. Such a clock is called an ***Astronomical clock*** and the time shown by it, ***sidereal time***. It should be remarked that 24 sidereal hours are equivalent to 23 hr. 56 min 4 sec of ordinary time called ***mean solar time***. Since the first point of Aries describes 360° on the equator in 24 sidereal hours, the sidereal time at any instant enables one to calculate the arc described by it westward from the meridian.

Let Fig. 4.5b represent the celestial sphere of the observatory. As usual let Z, P, EQ represent the zenith, the celestial pole and the equator respectively. Let σ be a celestial object' in

transit and let the simultaneous position of the first point of Aries be in the western hemisphere at γ. The arrowhead marks the eastern direction from γ. Obviously, R.A. of σ = arc $\gamma Q'$ and Decl. of σ = arc $\sigma Q'$.

The arc $\gamma Q'$ can be calculated from the time of transit of σ by the Astronomical clock, on the basis that 360° is equivalent to 24 hours. But as stated in SEC. 2.3, R.A. is usually measured in time directly, without conversion to arc. Hence the principle of measuring R.A. of a body is the following:

R.A. of a celestial body is equal to the sidereal time of its transit. ... ... (5.1a).

Decl. of σ = arc $\sigma Q' = ZQ' - Z\sigma = PN - Z\sigma$
since $PN = ZQ'$, each being the complement of ZP,
$= \varphi - z$, where φ and z are the latitude of the observatory and the Z.D. of σ respectively. .. ... (5.1b).

The instruments required for the measurement of R.A. and Decl. of a celestial body are therefore:

(1) A properly set astronomical clock, and
(2) An instrument which will enable one to judge just when a celestial body is on the meridian. Moreover it should be equipped with graduated circles for recording the meridian zenith distance of the body.

Such an instrument is called a *Transit instrument*. If emphasis is intended on the graduated circles, it is called a *Transit circle* or a *Declination circle*.

5. 2. It is easy to regulate a clock to show the period of a complete revolution of γ (or any star) as 24 hours; or rather to find out the rate at which it is gaining or losing over 24 hours so defined. But as γ is a geometrical point, not marked by any star, its transit cannot be directly observed and so the clock cannot be directly set to O hr. O min. O sec. We therefore proceed as follows:

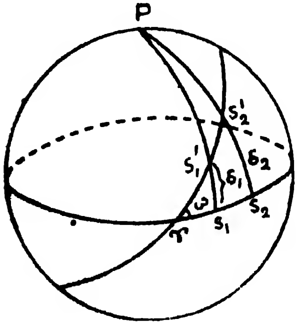

Fig. 5.2a

Let $\gamma S_1 S_2$ be the equator, $\gamma S'_1 S'_2$ the ecliptic, and S'_1, S'_2 two positions of the sun at the instants of transit on two different dates (Fig. 5.2a). Then PS'_1S_1 and PS'_2S_2 are the positions of the meridian on the two different dates at transits. The *interval* between the times of transit shown by the astronomical clock (though not properly set) will give us the arc $S_1S_2 = d$, say. Let $\gamma S_1 = x$; then $\gamma S_2 = x + d$. Let ω be the obliquity of the ecliptic and δ_1 and δ_2 the declination of the sun at the two observations. δ_1 and δ_2 can be found independently of the astronomical clock (See equation 5.1b).

Now from the right-angled spherical triangles $\gamma S_1 S'_1$ and

$\gamma S_2S'_2$, we can derive two equations involving the two unknown *quantities* ω and x; they therefore can be found. When x is known, the time which has elapsed since γ was on the meridian, is known; and so the astronomical clock can be set.

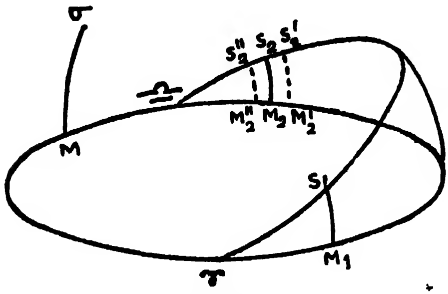

Fig 5 2b

Flamsteed's (First Astronomer Royal: 1646-1719) *method of setting the astronomical clock* is as folows:

Let γ M_1 Ω M_2 be the equator and $\gamma S_1 S_2 \Omega$ be the ecliptic. Let σ M be the secondary to the equator through the star σ so that its R A. = γ M = α hours, say The astronomical clock can be set when α is found

Let S_1, S_2 be two positions of the sun when his declinations $S_1 M_1$ and $S_2 M_2$ are equal. Let $\gamma M_1 = x$ hr ; then $\gamma M_2 = 12 - x$ hr , for the two spherical triangles $\gamma S_1 M_1$ and $\gamma S_2 M_2$ are identical and therefore γ $M_1 = \Omega$ M_2 .

Now the interval between the transits of S_1 and σ can be observed and so the equivalent of the arc M_1M in hours $- T_1$, say, can be found. Similarly let the equivalent of the arc M_2M in hours $- T_2$ We then have

$$T_1 = \gamma M - \gamma M_1 = \alpha - x \qquad (5.2a)$$

$$T_2 = \gamma M - \gamma M_2 = \alpha - (12 - x) \qquad (5.2b)$$

Adding, $T_1 + T_2 = 2\alpha - 12$, whence

$$\alpha = 6 + (T_1 + T_2)/2 \qquad (5.2c)$$

It should be noted that when σ lies between M_1 and M_2 the equation (5.2b) will be $T_2 = (12 - x) - \alpha$. The equation (5 2c) should then be accordingly modified.

It is not likely that the declination of the sun at the second transit should be exactly equal to that at S_1. In that case T_2 can be obtained as follows: Let δ_1 and δ_2, denoted in the figure by S'_2 M'_2 and S''_2 M''_2, be the declination of the sun at two successive transits, being a little greater and a little less than S_2 M_2. Let the corresponding intervals between the transits of the sun and the star be t_1 and t_2. Assuming that the change in declination of the sun is proportional to the change in his R A during the *short* period of a solar day we get

$$\frac{\delta_1 - \delta_2}{t_1 - t_2} = \frac{\delta_1 - \delta}{t_1 - T_2} \qquad (5.2d)$$

where $\delta = S_1 M_1 = S_2 M_2$.

Hence T_2 can be obtained and α follows from equation (5.2c).

A small correction is needed in equation (5.2b) on account of the fact that the first point of Aries moves backwards at a

small rate. The phenomenon is known as the ***Precession of Equinoxes*** and will be discussed in detail in CHAPTER 14. If the interval between the two observations (*i.e.*, when the sun is at S_1 and S_2) be expressed as a fraction of a year and p be the value of the precession per year, the R.A. of the star at the second observation should be $\alpha+\theta p$, instead of α. The modified value of α from the two equations then is given by

$$\alpha=6+\frac{T_1+T_2}{2}\ \frac{p}{2}\theta \text{ hours.} \qquad \ldots \qquad (5.2d)$$

The advantage of Flamsteed's method is that the actual values of the declinations at the two observations are not required, except for the small correction given by equation (5.2d). It is enough to observe that the declinations are equal. The actual values are difficult to obtain accurately from observed values on account of corrections involved. The first method however enables one to obtain the obliquity of the ecliptic as well as R.A. of the star, from the same set of observations.

5. 3. The *obliquity of the ecliptic* however may be obtained directly as follows: About the time of summer solstice, measure the declination of the sun for a few successive days. At first the declination increases, then remains almost steady for a few days and afterwards decreases. The maximum value attained is very nearly the obliquity required.

5. 4. We first describe briefly the construction and function of a *telescope* which is a constituent part of the transit instrument.

A telescope consists of a double convex lens fitted to the end of a straight metal tube. The lens is called the *Object glass,* because this end of the telescope is turned towards the object. For magnification of the image formed by the object glass an arrangement of smaller lenses is fitted to the other end and is called the *eye-piece*. There is a definite point inside the double convex lens such that any ray of light passing through it emerges unchanged in direction, while all other rays are bent towards the thicker portion of the lens. The position of the point can be worked out mathematically. It is called the *Optical centre* of the lens. Also there is a definite plane in which the image of any distant object (*i.e.*, an object from which rays of light entering the telescope are parallel) is formed. This plane is called the plane of the *Principal focus*.

Fig. 5.4a is the sketch of a telescope; AB is the object glass; E, the eye-piece; O, the optical centre, and FF′ the plane of the principal focus. A parallel beam of rays from a distant object in the direction FO converges to and forms an image at F in the principal

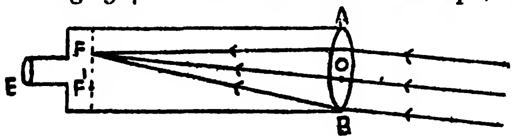

Fig. 5.4a

focal plane FF′; and the particular ray OF of the beam goes through unchanged in direction. The precise direction of the object is therefore the line joining the image and the optical centre of the object glass. Any other device of determining the direction of an object—say, a tube with cross-wires at the ends of a stretched thread—will be inconvenient and inaccurate in comparison. The essential point of having a telescope as a part of the transit instrument is to determine the direction of a celestial body with accuracy.

The *transit instrument* consists of a telescope TT rigidly attached to a perpendicular axis AA which ends in two equal cylindrical pivots. They rest on Y-shaped supports YY built on two piers PP of solid masonry work, so that the axis is horizontal (Fig. 5.4b). The pivot on its Y-shaped support, looked at longitudinally to the axis, is represented in the inset. The axis is thicker in the middle so that it may bear the weight of the telescope without bending. A great part of the weight of the instrument is balanced by lever arrangement LL; so the pivots press lightly on their supports, minimising the chance of unequal wearing out, which would upset the axis from its horizontal position.

Two graduated circles CC are rigidly attached to the telescope, one on each side, so that the axis AA is perpendicular to their planes and passes through their centres. On each side of the telescope six fixed and equally spaced microscopes are provided

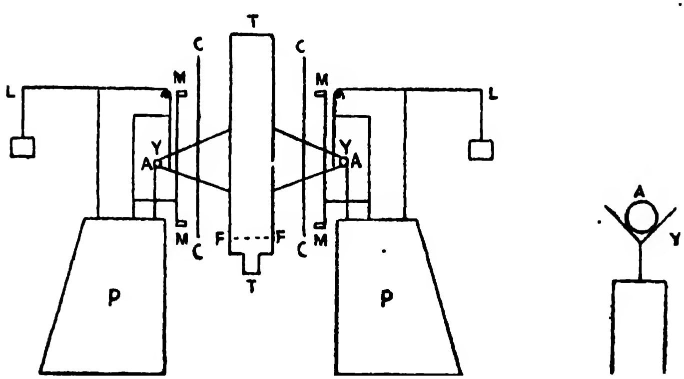

Fig. 5.4b

for reading the graduations of the circles. There is an additional fixed microscope of low magnifying power, called the ***Pointer,*** to read off degrees and larger sub-divisions. Subdivision of a degree is carried out to intervals of 5′ usually, finer graduation is both costly and liable to error.

In the plane of the principal focus of the object glass of the telescope TT there is a framework, shown separately in Fig. 5.4c,

on which an odd number—usually five—of vertical spider lines crossed centrally by two other horizontal ones are stretched at equal intervals. The framework and the real image of the celestial object observed are seen together, magnified by the eye-piece. While making observation, the path of the image of a star should lie midway between, and parallel to, the central lines. The framework is capable of rotational and slight lateral displacements for purposes of adjustment.

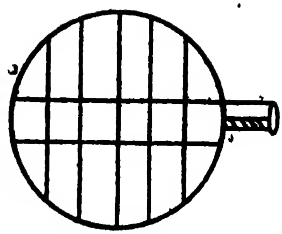

Fig. 5.4c

Definition: The line joining the optical centre of the object glass with the middle point of the middle vertical spider line of the transit telescope is called its *Line of collimation.*

5. 5. The transit instrument in perfect adjustment should be such that the line of collimation of the telescope is capable of being pointed anywhere on the meridian. The following conditions are clearly necessary to ensure this:

(1) The line of collimation should be strictly perpendicular to the axis. If it is not, there is an error called *Collimation error.*

(2) The axis should be accurately horizontal. If it is not, there is an error called *Level error.*

(3) The horizontal axis should point due east and west. If it does not, there is an error called *Deviation error.*

Adjustment for collimation error is usually done with the help of collimating telescope. In Fig. 5.5a, D, D′ are collimating telescopes (*i.e.*, ordinary small telescopes with cross-wires W, W′ at the focuses of their object glasses) one north and the other south of the Transit instrument, and facing each other. The collimating telescopes are first adjusted so that the image of the cross-wire of one coincides with the cross-wire of the other when viewed through the second. The adjustment can be carried out by uncovering apertures in the tube of the transit telescope. The lines joining the optical centre of the object glass to the cross-wire in each collimating telescope are therefore parallel. Let OE be the line of collimation of the transit telescope not accurately perpendicular to the axis XX′. Turn the telescope towards D, (E is the eye-piece and O, the object glass) and let the image of the cross-wire of D coincide with the middle point of the middle vertical

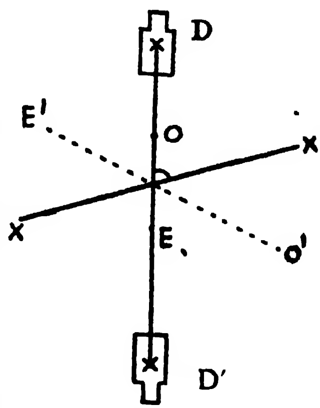

Fig. 5.5a

line of the telescope. OE is therefore parallel to the collimating lines of D and D′. (In the figure all lines are shown coincident for simplicity). Now turn the transit telescope south to D′; the new position of OE will be O′E′: The image of the cross-wire W′ will lie along EO and so will not be coincident with E′ the middle point of the middle vertical spider line. Adjustment is effected by shifting the frame-work carrying the spider line till the images of both the cross-wires are successively at the same distance from the centre of the frame-work and on the same side, left or right.

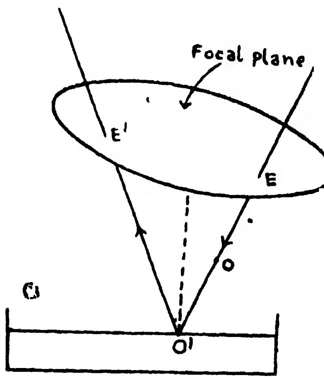

Fig. 5·5b

Collimation error having been previously corrected, *adjustment for level error* may be carried out as follows:

EO the line of collimation of the transit telescope is therefore perpendicular to the axis XX′, but suppose the axis is not horizontal. Turn the telescope downward on a trough of mercury. EO being not vertical, the ray EO from the middle point of the middle vertical spider line will be reflected from the horizontal surface of mercury along a different lint O′ E′ and form an image at a different point E′ Adjustment is made by raising one end of XX′ until the image coincides with the middle point itself.

The two adjustments having been carried out, *the Deviation error is adjusted as follows:*

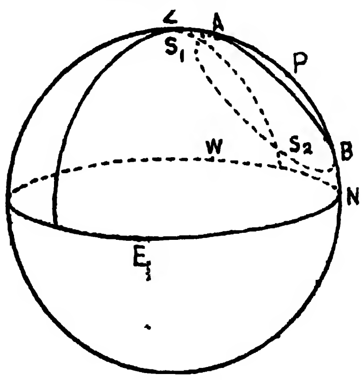

Fig. 5·5c

The previous adjustments make the axis horizontal and the line of collimation perpendicular to the axis. When therefore the telescope is turned about the axis, its line of collimation sweeps a vertical on the celestial sphere. Suppose the east end of the axis points a little north of the east. The vertical described by the line of collimation is represented by ZS_1S_2 in the adjoining figure (Fig. 5·5c). Let S_1 S_2 be a small circle described by a circumpolar star. The star can be viewed in the transit telescope at the two points S_1 and S_2. The time of passage of the star from S_1 to S_2 is shorter than the time from S_2 to S_1. The adjustment is made by screwing the east end of the axis *southward*, until the two intervals are equal.

Each adjustment is likely to upset the previous one slightly and so the process must be repeated. Even then perfect adjustment is impossible. Correction for residual errors however can be mathematically worked out and applied to the observed time of transit.

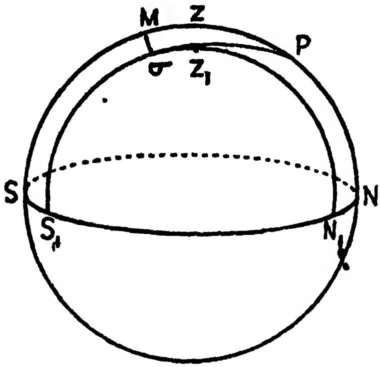

Fig. 5.5d

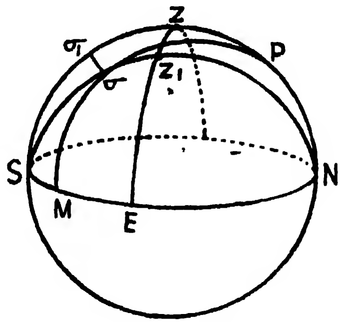

Fig. 5.5e

It may be noted that when collimation error alone is present the false meridian is a small circle parallel to the true meridian as represented in Fig. 5.5d. When level error alone is present the east end of the axis being lower than the west, the false meridian is a great circle through the north and the south points and east of the true meridian as represented in Fig. 5.5e. When deviation error alone is present the east end of the axis being a little to the north, the false meridian is a vertical as represented in Fig. 5.5c.

5. 6. The graduated circles attached to the transit instrument are for the purpose of measuring meridian zenith distances. The smallest graduations of the circle are at intervals of 5′. Degrees and multiples of 5′ are directly read in the Pointer. Odd minutes and seconds are however read on fine scales placed in the focal planes of the fixed microscopes. These scales have five divisions to an interval of 5′ on the graduated circle.

The image of the rim of the graduated circle gg′ is seen by the side of the fine scale ss′ in the field of view of the microscope (Fig. 5.6a). There is a circular hole O in the scale and a spider line xy, movable by a finely cut screw, stretches across it. The screw-head is divided into 60 equal parts and a complete rotation brings the spider line from one to the next graduation of the fine scale ss′. One division of the screw-head therefore corres-

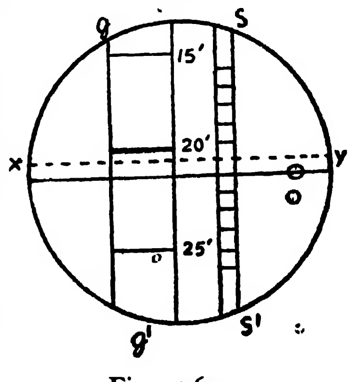

Fig. 5.6a

ponds to one second of arc. The scale is to be read at the position of the spider line through the centre of the hole O. In the figure the reading is a little over 20′; the whole number of additional minutes is one (from O to the dotted line). The fraction of a minute from the dotted line to the 20′ mark, is to be read on the screw-head. The spider line is first brought to coincidence with the dotted line and the number of divisions of the screw-head through which it has to be turned to move the spider line to the mark 20′ is noted. Suppose it happens to be 35.2; then the angle read is 1′ 35″.2.

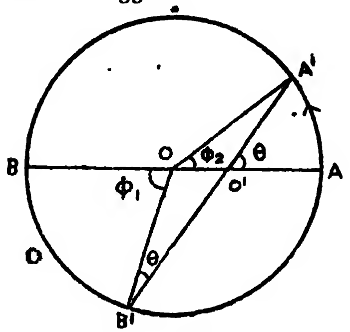

Fig. 5.6b

The average of the readings of the twelve microscopes joined to the reading of the pointer is the required consolidated reading of the graduated circles.

The reason for reading the circle at the opposite ends of a diameter is that the error caused by eccentric attachment of the circle can be eliminated in this way, as explained below.

Suppose O is the centre of the graduated circle (Fig. 5.6b) but the circle is so attached that it turns about O′. Let θ be the actual angle turned through by the circle, the points A and B occupying the positions A′ and B′. The reading at A will make out the angle to be $\angle$ AOA′ $= \varphi_2$; that at B, will make the angle to be $\angle$ BOB′ $= \varphi_1$. Join OA′ and OB′. Then $\angle$ OA′O′ $= \theta - \varphi_2$ and $\angle$O′B′O $= \varphi_1 - \theta$. Since these angles are equal, $\theta - \varphi_2 = \varphi_1 - \theta$.

$$\text{or} \qquad \theta = \frac{\varphi_1 + \varphi_2}{2}.$$

Hence the average of the angles obtained from the readings at the opposite ends gives the correct angle turned through by the circle.

5. 7. A celestial body is on the meridian when its image is just on the middle vertical spider line. The time of its transit is the time of its passage over this line. The image however moves rather rapidly in the field of view of the telescope and the chance is that an observer will be either too hasty or too tardy and seldom correct in noting the time of passage over the spider line. For this reason, the times of passages of the image over all the five lines are observed and the average is taken as the time of transit.

Formerly the times of passages of the image over the lines were recorded by what is known as the *eye and the ear method*. The observer first noted the hour and minute from the clock and began counting seconds by the ticks. While still counting, he looked into the telescope and recorded the times of passages on a piece of paper without taking his eye away from the telescope. Only a skilled observer could do the work with a fair degree of

accuracy. At present a mechanical device called the *chronograph* is employed for the purpose.

A piece of paper is wrapped round a cylinder CC which is made to revolve uniformly by a clockwork arrangement (Fig. 5.7). The cylinder has also a slow uniform lateral movement. A stylo pen S is attached to an iron lever, the tip resting against the cylinder, so that a continuous spiral line is marked on the paper as the cylinder rotates. The pendulum of a clock is provided with a metal brush at the lower extremity which dips into the mercury in a trough T when the pendulum comes to the vertical position. Immediately, an electric circuit from a battery B is completed through the mercury, pendulum and back to the battery. There is an electro-magnet M in the circuit which is excited and draws the stylo pen aside. As the pendulum moves away, the circuit is broken and the stylo pen falls back to its original position, drawn by a spring R. Thus a dentation mark is left on the paper at each swing of the pendulum. The electro-magnet can also be excited, independently of the pendulum, by pressing a key K, as shown in the figure.

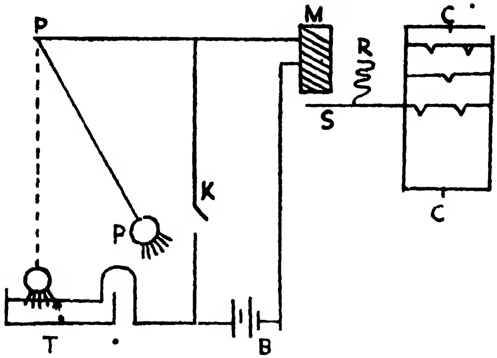

Fig 5 7

Time by the clock corresponding to a particular dentation mark is written down on the paper at the start. The observer then has merely to press the key K at the passage of the image over each spider line. On unwrapping the paper, the times of passage over each spider line can be figured out.

5. 8. The declination of a celestial object is given by the equation (5.1b). The meridian zenith distance z is one of the quantities involved in it; and obviously it is the difference of the readings of the graduated circle when the telescope is turned first to the object and then to the zenith. But the reading for the zenith must be obtained indirectly; for the zenith is a geometrical point and cannot be viewed in the telescope. Suppose the telescope is turned down on a mercury trough and adjusted till the middle point of the middle vertical spider line coincides with its own image. The line of collimation of the telescope then points to the Nadir. Take the reading of the graduated circle and subtract 180° from it; the difference gives the reading for the zenith.

The other quantity involved in the equation (5.1b) is the latitude of the observatory; it is obtained as follows. Here however we

do not take into account certain corrections, explained in the next chapter, due to the fact that a ray of light undergoes refraction in passing through the atmosphere of the earth.

In Fig. 5.5c, let A and B be the positions of a circumpolar star at the upper and the lower transit respectively. Let

z_1 = zenith distance of B = arc ZB = ZP + PB

z_2 = zenith distance of A = arc ZA = ZP - PA

$z_1 + z_2 = 2\,ZP = 2\,(90° - PN)$; for PA = PB, each being the north polar distance of the same star.

Now PN = the latitude of the place = φ, say.

$z_1 + z_2 = 2\,(90° - \varphi)$, whence

$$\varphi = 90° - \frac{z_1 + z_2}{2} \qquad \ldots \qquad \ldots \qquad (5.8).$$

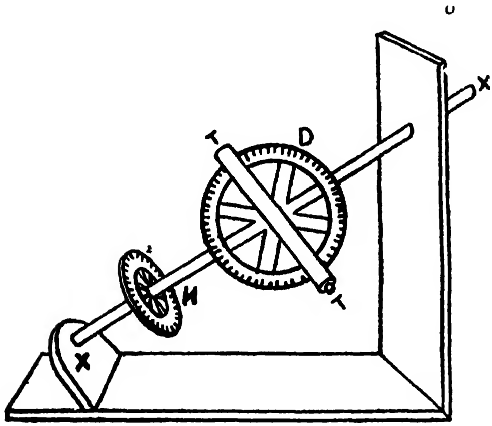

Fig. 5.9

5. 9. To measure accurately the angle subtended at the eye by two close stars, it is necessary that (1) they should remain in the field of view of the telescope for a considerable time and (2) the magnification should be fairly large. These requirements are provided in the instrument called the *Equatorial*.

The equatorial is a telescope TT (Fig. 5.9) with a large object glass and high magnifying power. It is attached to an axis XX, pointing towards the celestial pole in such a way that it can rotate in a plane containing the axis: the axis is called the ***Polar axis***. There is a graduated circle D, called the ***declination circle***, to measure the angle through which the telescope is turned from the polar axis. The whole instrument turns about the polar axis and there is a second graduated circle H, whose plane is perpendicular to the axis, in order to measure the angle turned through. This circle is called the ***Hour circle***.

The instrument can be kept rotating about the polar axis at any desired rate with the help of a clock-work arrangement; and it is possible to keep a celestial object fixed in the field of view.

5. 10. The angular distance between two close objects is measured with a ***Micrometer***, placed in the focal plane of the equatorial. The micrometer consists of two forks FF having a spider line stretched across the prongs of each. The forks are

movable, one sliding inside the other, by accurately turned screws SS (Fig. 5.10a) A complete rotation of the screw brings the spider line from one graduation to the next of a fine scale CC placed inside. The screw-head is divided into 100 equal parts so that the distance between the spider lines can be read to two places of decimals in terms of the scale. The micrometer can be rotated in its plane so that CC may be made to lie along the line joining any two stars under observation. Each spider line is made to pass through the image of a star; and the distance between them is read on the scale and the screw-heads.

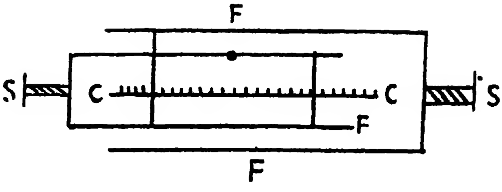

Fig. 5.10a

The angular value of each division of the scale is found as follows Separate the spider lines by a known number of divisions of the scale, say n. Turn the equatorial to a star of declination δ and stop the clock work; the image of the star moves across the field. Adjust the micrometer so that the path is along the scale CC. Let t be the time taken by the star to move from one spider line to the other.

Let BB′ (Fig. 5.10b) represent the arc of a small circle described by the star in t sidereal hours. The angle subtended by the arc at its centre O′=15t degrees. The length of the corresponding arc AA′ of the parallel great circle subtending an equal angle at its centre O, is given by

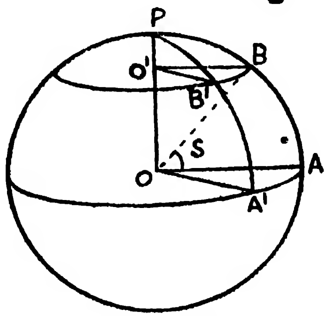

Fig. 5.10b

$$\frac{\text{arc } AA'}{\text{arc } BB'} = \frac{\text{radius of the great circle}}{\text{radius of the small circle}} = \frac{1}{\cos\delta}$$

$$\text{arc } AA' = \text{arc } BB' \sec\delta$$

$= n \sec\delta$ in terms of the scale of the micrometer.

But arc AA′ = 15t degrees. Hence the equivalent of one division of the scale = $15t \div (n \sec\delta)$ degrees = $15t \cos\delta / n$ degrees.

Example Worked Out

The graduated circle of a transit instrument is attached to the axis at a point distant c from the centre. If 2r be the diameter of the circle, calculate the maximum error when the circle is read at one point only.

Let O be the centre and O′ the point of attachment of the circle (Fig. 5 6b) Let OO′A be a radius. Let the circle be turned through an angle θ, so that A goes to A′ If the angle is read at A, it will appear to be $\angle AO'A'$, while the actual angle turned through is AOA′ e, the error, is $- \angle OA'O'$. From the triangle OA′O′,

$$\frac{\text{Sin}\, e}{c} = \frac{\text{Sin}\, \theta}{r}$$

Or, $\sin e = c\,\dfrac{\sin\theta}{r}$. Therefore the maximum value of e is given by $\sin e = \dfrac{c}{r}$.

Exercise 5

1 Explain Flamsteed's method of setting the Astronomical clock What are its special advantages?

2 Describe the Transit instrument

3 Calculate the error in the time of transit of a star on the equator, if the level error be 2″ the latitude of the observer being O°. Calculate the error when the collimation error is 2″ in the same case

4 If t be the interval between the passages of a star on the equator over two consecutive spider lines of a transit instrument, calculate the interval for a star of declination 60°

5 If the azimuth error of a transit instrument be 2″, find the error in the time of transit of a star on the equator the latitude of the observer being 60°

6 REFRACTION

6.1. The earth is surrounded by air which is transparent, compressible and heavy. The density of air in layers concentric with the surface of the earth decreases gradually with the height of the layer. For, the lower a layer is, the more it is compressed, having to bear the weight of a larger mass of air above.

Consider a ray of light SA coming from a star S (Fig. 6.1). It gets bent towards the normal in passing from a rarer medium above to a denser medium below. It is therefore bent more and more as it enters lower and lower layers of atmosphere and finally reaches the observer on the surface of the earth at O in the direction S′O. OS′ is the direction in which the star is seen while the true direction is OS, parallel to AS. The angle SOS′ between the apparent and true directions is called *Refraction.*

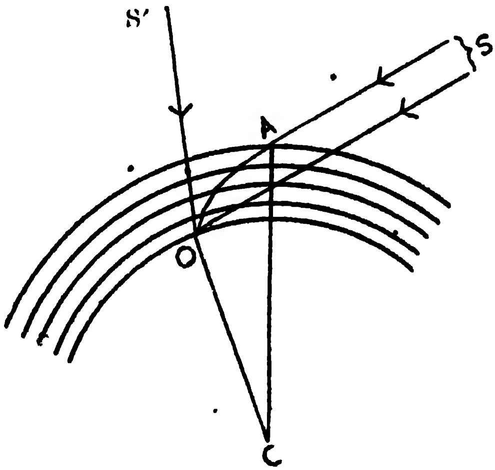

Fig. 6.1

In the last chapter, elaborate arrangements have been described to locate the true direction of a celestial body It will now be seen that they are futile unless the amount of Refraction can be estimated.

6.2. Consider a star *not very far from the zenith*. In Fig. 6.1, let the outermost circle represent the extreme boundary at which the atmosphere produces any appreciable refraction. Let a ray of light from a star S meet this boundary at A and being bent gradually reach the observer at O. Let C be the centre of the earth. For the purpose of estimating the amount of bending we are concerned only with the sector of the atmosphere of angle OCA, which should be small if the height of the boundary be small compared with the radius of the earth.

An estimate of the height of the boundary can be made from observations on meteors. A meteor bursts into light at the

point where it rushes into atmosphere, on account of the heat generated by friction of air. The height of this point can be ascertained thus:

Let two observers A and B, separated by a known distance AB, note the directions of the point M where a meteor bursts into light. The side AB and the angles MAB and MBA of the triangle MAB are therefore known and so the height of M above the ground can be found. From such observations on meteors it has been ascertained that the height of *the effective atmosphere* does not exceed 150 miles at the most. This however is small compared with the radius of the earth.

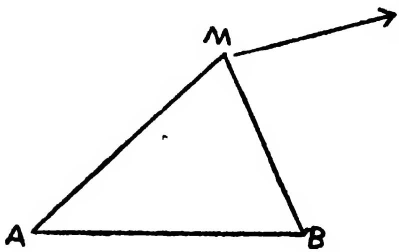

Fig. 6.2a

It follows that the angle OCA of Fig 6.1 is small and the layers of atmosphere within the small sector OCA may be regarded as *parallel planes.*

Consider a ray of light ABCDE (Fig. 6.2b) entering, through successive plane layers of any refracting media of different densities, into the lowest in which let DE be its direction. Consider a parallel ray A′D′ directly entering the lowest layer in which let D′E′ be its direction. It is known experimentally that DE and D′E′ are parallel. We therefore need consider only the lowest layer of air in calculating atmospheric refraction.

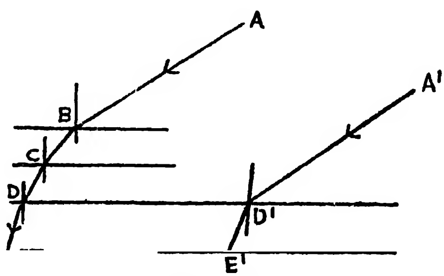

Fig. 6.2b

In Fig. 6.2c, the parallel horizontal lines represent the boundaries of the lowest layer of atmosphere.

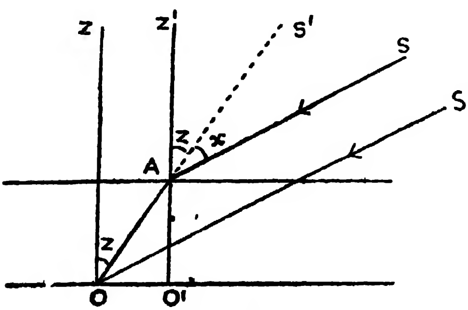

Fig. 6.2c

Let a ray of light SA from a star S be refracted on entering the

layer along AO and reach the observer O. Draw the vertical line OZ. The apparent zenith distance of the star is $\angle ZOA = z$, say. Produce OA to S′ and let O′AZ′ be parallel to OZ. Refraction which is the angle between the apparent and true directions of the star is equal to $\angle SOS'$; let it be denoted by x. Obviously $\angle SAS' = x$, $\angle S'AZ' = z$, $\angle OAO' = z$. Hence the angle of incidence of the ray $\angle Z'AS$ is $z+x$ and the angle of refraction $\angle OAO'$ is z.

$\therefore \dfrac{\sin(z+x)}{\sin z} = \mu$, where μ is the refractive index from vacuum to the lowest layer of atmosphere.

Or, $\sin z \cos x + \cos z \sin x = \mu \sin z$.

Or, $\sin z + x \cos z = \mu \sin z$, to the first order of the small quantity x, which is measured in radians.

$$\therefore x = (\mu - 1) \tan z \quad \ldots \quad \ldots \quad \ldots \quad (6.2a)$$

Since $206265'' = 1$ radian, the refraction r in seconds of arc is given by $r'' = 206265'' \ (\mu - 1) \tan z = k'' \tan z \ \ldots \quad \ldots \quad (6.2b)$
The formula, it should be remembered, has been derived on the understanding that z is not very large.

It will be seen from Fig 6.2c, that a celestial object is raised towards the zenith on account of refraction. The true zenith distance is therefore obtained by adding the amount of refraction, given by equation (6.2b), to the apparent zenith distance. True altitude is got by subtracting the correction from the observed altitude. It should also be noticed that the azimuth is not affected by refraction; for the displacement takes place along the vertical through the body.

6.3. The constant k of equation (6.2b) is called the *coefficient of refraction.* Its value is better derived from astronomical observation rather than by laboratory determination of μ.

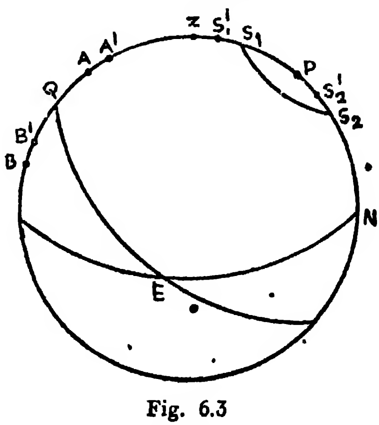

Fig. 6.3

Let S_1 and S_2 (Fig. 6.3) represent the true positions of a circumpolar star at upper and lower transits. On account of refraction, the apparent positions will be S'_1 and S'_2 respectively. Let z_1 and z_2 be the observed zenith distances. Then,

$ZS_1 = z_1 + k \tan z_1$

Or $ZP - PS_1 = z_1 + k \tan z_1$

$ZS_2 = z_2 + k \tan z_2$

Or $ZP + PS_2 = z_2 + k \tan z_2$

Now $PS_1 = PS_2$. Therefore, adding—

$$2ZP = 2(90^\circ - \varphi) = z_1 + z_2 + k (\tan z_1 + \tan z_2) \ldots (6.3a)$$

where φ is the latitude of the place of observation. If φ be

known, k can be obtained from the equation above; for z_1 and z_2 are known from observation. If φ be also unknown, observation on a second circumpolar star yields a similar equation:

$$2(90^\circ - \varphi) = z_3 + z_4 + k(\tan z_3 + \tan z_4), \text{ say.} \quad \ldots \quad (6.3b)$$

Solving the two equations, we get

$$k = \frac{z_1 + z_2 - z_3 - z_4}{\tan z_3 + \tan z_4 - \tan z_1 - \tan z_2} \quad \ldots \quad \ldots \quad (6.3c)$$

and $$\varphi = 90^\circ - \frac{(z_1 + z_2)(\tan z_3 + \tan z_4) - (z_3 + z_4)(\tan z_1 + \tan z_2)}{2(\tan z_3 + \tan z_4 - \tan z_1 - \tan z_2)} \quad \ldots \quad (6.3d)$$

Equation (5.8) should be compared with the equation above to see how it is modified when correction for refraction is taken into account.

In practice we need not depend upon only two observations to get k and φ. A number of observations on a number of circumpolar stars may be pooled together by a method known as the *method of Least Squares* to yield more reliable results

The value of k is 58″.2 The equation (6.2b) gives a fairly accurate measure of refraction up to about 70° of zenith distance. Change of temperature and atmospheric pressure should further be taken into account to obtain greater refinement. Refraction at greater zenith distances may be known from direct observation: the Z.D. of a star at any instant can be calculated and compared with the observed value.

Bradley's (English Astronomer: 1692-1742) method of obtaining coefficient of refraction is to observe transits of the sun at summer and winter solstices, instead of that of a second circumpolar star.

In Fig. 6.3, let A and B be the true positions of the sun in transit at summer and winter solstices respectively; and A′ and B′ their apparent positions. Let s and s′ be the zenith distances for the positions A′ and B′. Then

$$ZA = s + k \tan s, \text{ i.e., } ZQ - AQ = s + k \tan s;$$
$$ZB = s' + k \tan s', \text{ i.e., } ZQ + BQ = s' + k \tan s', \text{ where}$$

Q is the point where the equator meets the meridian.

$$2ZQ = s + s' + k(\tan s + \tan s');$$

for AQ = BQ = the obliquity of the ecliptic. Now ZQ = the complement of ZP = PN = φ, the latitude of the place. Hence

$$2\varphi = s + s' + k(\tan s + \tan s') \quad \ldots \quad \ldots \quad (6.3c)$$

If φ be known, the equation at once gives k. Otherwise it has to be combined with an equation of type (6.3a) for a circumpolar star, and both k and φ can be found.

The advantage of Bradley's method is that the coefficient of refraction can be found in low latitudes where the lower transit

of a circumpolar star may not fall within the limits of our formula. The disadvantage is that observations are to be made at an interval of six months when atmospheric conditions do not remain identical. It will be obvious that if a catalogue of known stars be at hand, the problem in any latitude is quite simple.

6.4. Refraction of a body on the horizon is about 30′. Since the angular diameter of the sun and the moon are also of this magnitude, they are really completely below the horizon when they appear just above it.

Another effect of refraction is that the discs of the sun and the moon appear elliptic when near the horizon. Even a small change of 30′ in Z.D. near the horizon causes a change of about 4′ in refraction. The lower limb of the sun or the moon is therefore raised 4′ more than the upper. The vertical diameter is therefore considerably shortened; but the horizontal diameter remains unaffected. So the disc appears elliptic.

A curious effect of refraction may occasionally be observed during a lunar eclipse: the sun and the moon may both be just above the horizon while a lunar eclipse is on.

6.5. *Cassini's formula for refraction*: Cassini derived a formula for refraction on the hypothesis that the atmosphere is spherical but homogeneous; the formula is found to give good results as far as 80° of Z.D.

Let a ray of light ABO from a star enter the earth's homogeneous atmosphere at B and on refraction proceed in a straight line to the observer O. Let C be the centre of the earth. Then CBZ′ is normal to the spherical surface of the atmosphere at B, and COZ is the direction of the zenith at O. Let AB be produced to meet OZ at O′; also produce OB to B′. Then the apparent Z.D. of the star is $\angle ZOB = z$, say; the refraction is $\angle OBO' = x = \angle ABB'$. Let $\angle CBO = z'$. Then

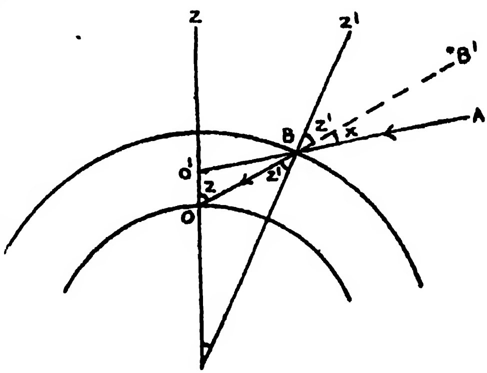

Fig. 6.5

$\sin (z'+x)/\sin z' = \mu$. Hence as before $x = (\mu - 1) \tan z'$.

To express z' in terms of z, we see from the triangle OBC that $\dfrac{\sin z'}{\sin z} = \dfrac{r}{r+h} = \dfrac{1}{1+n}$ where r = the earth's radius and h is the height of the atmosphere, and $n = h/r$. It follows that

$\sin z' = \sin z/(1+n)$. Therefore, $\cos^2 z' = 1 - \sin^2 z/(1+n)^2$
$= (\cos^2 z + 2n)/(1+n)^2$, neglecting n^2 which is small.

$$\therefore \tan z' = \frac{\sin z/(1+n)}{\cos z \sqrt{(1+2n \sec^2 z)/(1+n)}}$$

$$= \tan z(1 - n \sec^2 z)$$

$\therefore x = (\mu - 1) \tan z(1 - n \sec^2 z)$ which is Cassini's formula. It is easily seen that it can be put in the following form

$$x = A \tan z + B \tan^3 z.$$

Examples worked out

1. If α and α' be the true and apparent altitudes of a body as affected by refraction, show that $\alpha = \alpha' - k \cot \alpha'$, where k is the coefficient of refraction.

The true Z.D. $= 90^\circ - \alpha$

The apparent Z.D $= 90^\circ - \alpha'$

$\therefore 90^\circ - \alpha = 90^\circ - \alpha' + k \tan (90^\circ - \alpha') = 90^\circ - \alpha' + k \cot \alpha'$

i.e. $\alpha = \alpha' - k \cot \alpha'$.

2. Prove that the polar distance of a star is unaffected by refraction when its azimuth measured from the north point is the maximum. (Andhra '42 Part III).

Hint: The polar distance will be unaffected when refraction causes a change of position perpendicular to the polar distance *i.e.* when the vertical through the star touches its diurnal path. Therefore the azimuth from the north point is the maximum.

Exercise 6

1. How will you determine the coefficient of refraction at a place of low latitude? How will you determine it at large zenith distances?

2. Calculate the angle subtended at the centre of the earth by the length of the tangent at a point of the earth terminated by the boundary of effective atmosphere. (Take the radius of the earth to be 4000 miles and the effective height of the atmosphere to be 50 miles).

3. Twilight is caused by sun's rays reaching the atmosphere and illumining the dust particles suspended in it. If twilight lasts till the sun is 18° below the horizon, estimate the height of the atmosphere. (Take the radius of the earth to be 4000 miles and cos 9° = .98796).

4. The meridian altitudes of a circumpolar star are observed to be 30° and 60°. Find the latitude of the place, applying correction for refraction. (Coefficient of refraction = 58″.2).

5. The apparent Z.D. of a star is given by $5 \sin^2 z + 7 \sin z - 6 = 0$. Calculate the true Z.D., taking the coefficient of refraction as 58″.2 (C.U. 1920).

6. Let z_1 and z_2, each less than 50°, be the observed meridian Z.D. of a circumpolar star at a place of latitude φ. Calculate the index of refraction of air at the earth's surface.

7 THE SUN

7.1. Diurnal motion of stars having been completely explained by the rotation of the earth about its axis, it remains to explain only the motions of celestial bodies relative to stars. The motion of the sun relative to stars has been described in SEC. 2.1. It is on a great circle called the ecliptic and the period of a complete revolution is a year.

The angular diameter of the sun is found to vary at different points of the ecliptic. This is undoubtedly due to variation of his distance from the earth.

Let S be the sun and O the observer (Fig. 7.1a). Draw tangents to the circle, representing the sun, from the observer O. Let r be the distance of the sun's centre from O, θ the angle between the tangents *i.e.* the angular diameter of the sun and d the linear diameter of the sun. Since θ is small, supposing it to be expressed in radians,

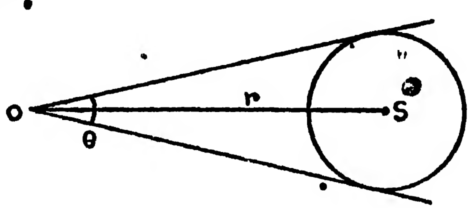

Fig. 7.1a

$$r\theta = d\text{; or } r = \frac{d}{\theta}\text{; } i.e.\ r \text{ varies as } \frac{1}{\theta}\text{, as d is constant}$$

The apparent path of the sun on the celestial sphere is only a projection along the line of sight. Its true shape is determined

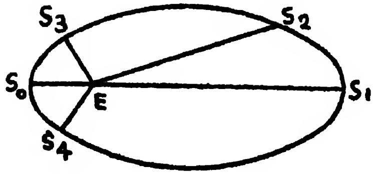

Fig. 7.1b

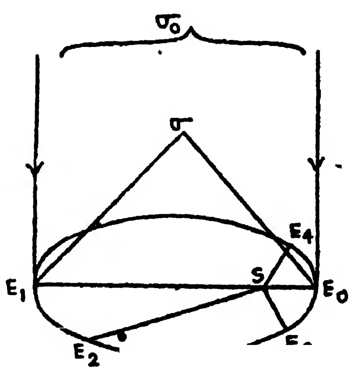

Fig. 7.1c

as follows: Let the direction of the sun on a particular day be

ES_1 (Fig. 7.1b). Cut off the length ES_1 inversely proportional to the observed angular diameter of the sun. Then S_1 represents the position of the sun for the particular day. In the same way plot positions of the sun throughout the year. The path of the sun so traced out is found to be an ellipse of small eccentricity with the earth at one of the *focuses*. The sun is nearest the earth on the 31st. December; he is then at an end of the major axis of the ellipse, which is called the *perigee*; he is farthest when he is at the opposite end on the 1st July and this point of the path is called the *apogee*.

The question however remains whether the sun actually moves in an ellipse with the earth at a focus, or the earth moves in an ellipse with the sun at a focus. For the appearance will be the same to an observer on the earth in either case. The two hypotheses are represented in Figs. 7.1b and 7.1c respectively. The positions of the earth E_1, E_2, E_3, E_4 in Fig. 7.1c correspond to the positions of the sun S_1, S_2, S_3, S_4 in Fig. 7.1b. Direction as well as distance of the sun as seen from the earth are the same in both the figures for any corresponding situation. If we regard the earth to be revolving round the sun, the point of its orbit nearest the sun is called the *perihelion* and the point farthest the *aphelion*.

The eccentricity of the elliptic path being quite small, we shall generally take it to be circular for simplicity.

7.2. Ptolemy (Alexandrian Astronomer: A.D. 140) supposed that the sun actually revolved round the earth and his hypothesis remained current for about fourteen centuries after him. Copernicus (Polish Astronomer: 1473 – 1543) first put forward the hypothesis that the earth revolves in a circle round the sun. It was at first challenged on the ground that if the earth really moved round the sun, the enormous shift of the earth in six months, say from E_1 to E_0 in Fig. 7.1c, would cause an appreciable change in the direction of any star; but no such change was then appreciable. Copernicus replied that this only showed that distances of stars were enormous. Later discoveries proved him to be correct

Decisive facts in support of the Copernican hypothesis are the following:

(1) Changes in the observed directions of a few stars in course of six months, such as would be expected from the shifting of the position of the earth—*stellar parallax* as it is called—have since been detected.

(2) After Copernican hypothesis had been put forward, there was persistent search for stellar parallaxes to secure evidence for its correctness. In course of such search, Bradley discovered

a different phenomenon, known as *aberration* of light, which provided a new evidence of earth's motion in an orbit. Aberration will be treated fully in a subsequent chapter (CHAPTER 14). Here we only mention that the velocity of light being known to be finite, the direction in which light from a star seems to reach an observer on the earth will be one if the earth be at rest and another if it be in motion. Should the earth move round the sun, the relative direction of light from a star too will go through a periodic change. Such periodic change was discovered by Bradley. It therefore provides an indirect proof of the earth's revolution round the sun.

(3) Evidence of the earth's orbital motion has also been obtained by the spectroscopic method. In the spectrum of a star, there are usually dark lines, due to absorption of the star's light by the outer atmosphere of the star. These lines have specified wave lengths depending on the elements existing in the atmosphere. Consider such a dark line in the spectrum of the star σ_0 (Fig. 7.1c), in the plane of the earth's orbit. Supposing the earth to be moving in the direction of the arrowhead, at E_0 the observer should be moving towards the star. Consequently, the number of light waves received by him per second increases and according to Doppler's principle the spectral lines of the star shift towards the violet end of the spectrum. Six months later the earth is at E_1 and moving away from the star. There should now be a shift of the same spectral lines towards the red end of the spectrum. Photograph of such spectra with a comparison spectrum actually reveals the shift and so demonstrates the earth's motion. It should be mentioned further that it is possible to calculate the earth's velocity from the amount of the shift.

7.3. When the sun's disc is observed in a telescope (with a dark eye-piece) or when the telescope is so arranged that a real image of the disc is cast on a white card-board, dark spots of irregular shapes are seen on it, particularly in the central portions. The spots move from the east end of the disc to the west. They usually change in shape and sometimes altogether disappear; but it is generally possible to observe their period of revolution. They take about 13½ days to cross from one end of the disc to the other, remain out of view for an equal period of time and reappear once again at the east end.

An external object coursing round the sun will appear as a dark spot against his bright disc. But the fact that the dark spots observed on the sun's disc are not caused by any external objects can be seen in the following way.

In Fig. 7.3a, let the inner circle, centre S, represent the sun and the outer circle ACBD, the path of any supposed body,

external to the sun. The tangents to the sun from an observer on the earth are almost parallel and are indicated in the figure.

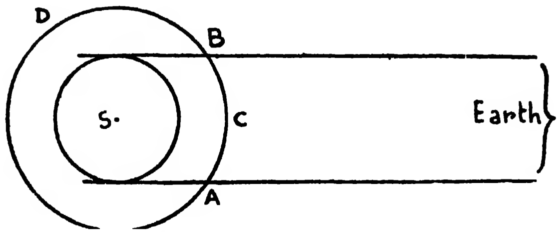

Fig. 7.3a

The body will be seen against the disc of the sun as a dark spot while it describes the comparatively shorter arc ACB. While it describes the remainder of the orbit BDA it will not be projected as a dark spot; nor will it be visible on account of the brightness of the sun or on account of its passage behind the sun And it would be extremely unlikely that equal times should be taken to describe such widely unequal portions of the orbit. The spots therefore cannot be due to bodies external to the sun and must be objects on the surface of the sun. Their motion across the disc of the sun must be due to the rotation of the sun about an axis; for there could be no other simple explanation why they move in the same regular way.

The observed period is about 27 days. This however is relative; for the earth is also moving round the sun in the *same direction*. The true period of rotation of the sun is found as follows:

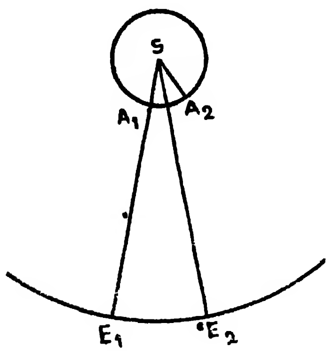

Fig. 7 3b

In Fig. 7.3b, the inner circle A_1A_2, centre S, represents the sun and the outer circle E_1E_2, the orbit of the earth. Let A_1 be a spot and E_1 a position of the earth and let SA_1E_1 be a straight line. Let A_1 move to A_2 and E_1 to E_2 in course of a day. Let

T = the observed *i.e.*, the relative period of motion of the spot in days.

S = the true period in days.

Y = the number of days in a year.

The average angle gained over the earth by the spot in a day = 360°

Also $\angle E_1SE_2 = 360°/Y$ and $\angle A_1SA_2 = 360°/S$. Hence the

average daily gain is also $=(360^\circ/S)-(360^\circ/Y)$. Therefore,

$$(360^\circ/S)-(360^\circ/Y)=(360^\circ/T),$$

$$\text{or,} \quad (1/S)-(1/Y)=1/T \quad \ldots \quad (7.3)$$

Since Y and T are known, S is obtained; it is found to be 25 days, nearly.

It may be noted, in passing, that rotation of the sun about an axis being admitted, his shape must be spherical since his disc always appears circular.

Example worked out

Describe diurnal phenomena at different latitudes, if the axis of the earth were (i) perpendicular to, (ii) in the plane of, the ecliptic. (Dac. U. '35, C.U. '34).

Diurnal phenomena, as regards stars, will obviously be as explained in SEC. 4 3, in both the cases, those depending on the sun are explained below.

Case (i) By the condition of the problem, the ecliptic coincides with the equator. The diurnal path of the sun always coincides with the equator Hence, as explained in SEC. 4 3, duration of day will be equal to duration of night all over the earth, and at all times: there will be perpetual equinox. At the poles, the sun will be continually on the horizon; there will be perpetual sun-rise. There will be no variation of seasons, because the sun describes the same diurnal path, namely the equator, all through the year. Places of high latitudes will get cooler and cooler and equatorial regions will get hotter and hotter with time.

Case (ii). The ecliptic is a secondary to the equator in this case. As explained in SEC. 4.3, at the equator of the earth, days and nights will be equal in duration throughout the year; at intermediate latitudes days will be longer during half the year and shorter during the other half; at the poles there will be day of six months' duration and night of equal duration. Changes of seasons at intermediate latitudes will generally be as explained in SEC. 4.4, but extremes of temperature will be more accentuated. Midnight sun, and days and nights of more than 24 hr. will occur at all intermediate latitudes. At the poles temperatures will in general be higher than what we have actually; because during the period of perpetual day the sun will gradually attain the highest altitude 90°.

Exercise 7

1. Find the diameter of the sun in miles, given that his angular diameter is 32′ and his distance is 93000000 miles.

2. How long would it take to fly round the sun at 2 miles a minute? (Distance of the sun is 93000000 miles and the angular diameter is 32′).

3. The apparent diameter of the sun when least is 31′ 32″ and when greatest 32′ 36″. Calculate the eccentricity of the earth's orbit. (C.U. 1948).

4. Find the velocity of the earth in its orbit, supposing the orbit to be circular and of radius 93000000 miles.

5. Venus moves round the sun in the same direction as the earth in .62 years. What would be the apparent period of solar spots to an observer on Venus, if it be 27 days to an observer on the earth?

6. The sun's angular diameter is observed to be 31′ 32″ at midsummer and 32′ 36″ at midwinter. Show that the sun is farther at midsummer than at midwinter in the proportion of 31 to 30 approximately. Why then is winter colder though the sun is nearer? (C.U. 1943).

8. THE PLANETS

8.1. Motions of planets relative to stars have been described in a general way in SEC. 2.1. The main features are: (1) at times a planet moves in the same direction among stars as the sun and the motion is then called *Direct*, (2) at times it moves in a direction opposite to that of the sun, when the motion is called *Retrograde* and (3) at times it seems to have no motion relative to stars when the planet is said to be *Stationary*. The actual paths mapped out on the background of stars show curious patterns represented in Fig. 8.1a.

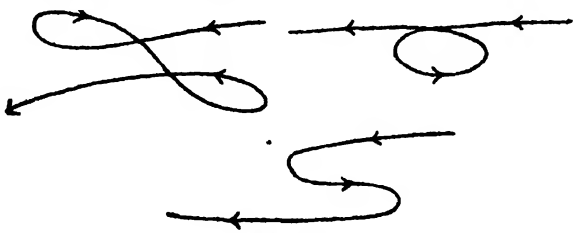

Fig. 8.1a

Any theory of planetary motions, to be acceptable, should yield apparent motions of planets in agreement with observations described above.

Among ancient theories, the most important is the Ptolemaic put forward in the second century after Christ. According to it, the spherical earth is fixed at the centre of the entire system. The sun and the moon revolve round it in circles. The planets Mercury and Venus move in circles called *Epicycles*, the centres of which move round the earth in circles called *Deferents*. The deferent of Mercury is inside that of Venus; and the deferents of both are inside the orbit of the sun (Fig. 8.1b). Moreover, the centres of the two epicycles of Mercury and Venus are always on the line joining the sun and the earth.

The motions of Mars, Jupiter and Saturn are similarly a combination of motions of the planets in their respective epicycles and the motions of the centres of the epicycles in their

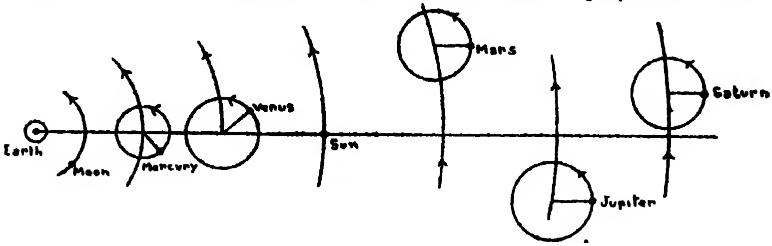

Fig. 8 1b

respective deferents. One additional point with regard to these planets is that the lines joining the planets to the centres of the epicycles are always parallel to the line joining the earth and the sun.

Planets are thought to be situated on crystalline spheres, stars on another which encloses all the rest. Diurnal motion is supposed to be produced by the rotation of these spheres about the earth's axis.

This theory had been supreme for about fourteen centuries until Copernicus proposed a new one. The modern theory of planetary motion is really the Copernican, with slight modifications. With progress of time the Ptolemaic theory had to be modified by adding epicycles on epicycles to produce agreement with observations, until it became much too complicated. The Copernican theory (represented in Fig. 8.1c) which places the sun at the centre and makes all the planets, including the earth revolve round him in circles, is much simpler in comparison Diurnal motion, on this theory, is explained by the rotation of the earth about its axis. But Copernicus too supposed the orbits to be *circular*. In particular cases, he placed the sun a little out of centre or introduced a much smaller number

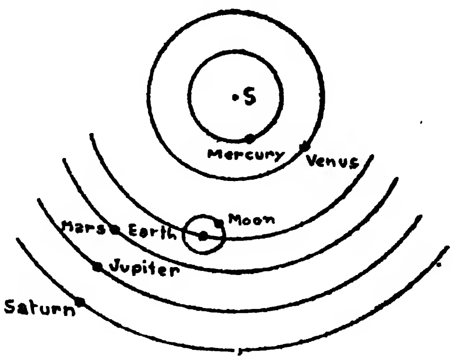

Fig. 8.1c

of epicycles to bring agreement with observation. The ancients believed the circle to be the perfect curve and celestial motions,

they thought, must be composed of circular motions. We shall explain later how Kepler (German Astronomer: 1571-1630) was able to construct the path of a planet directly from observation and found it to be an ellipse.

It should be remarked that neither Ptolemy nor Copernicus possessed facts which could form any solid basis of their theories. The Copernican theory was virtually accepted before all objections to it could be overcome by means of observation. We however give below steps which would logically lead to it, though they are not of historical order.

8.2. That planets show phases like the moon was not known before the time of Galileo (Italian Astronomer (1564-1642). He discovered the phenomenon with his newly invented telescope. Occurrence of phases show that planets are not self-luminous, the illumination observed is due to sun-rays. Moreover, that they are spherical bodies rotating about axes through their centres can be established by the methods of sec. 7.8.

Let the sphere, centre O, represent a planet, OS the direction of the sun and OE that of the earth (Fig 8.2). The portion illuminated by the sun is bounded by the circle ABA′, perpendicular to OS, the portion visible to the observer on the earth is bounded by the circle ADA′, perpendicular to OE. The illuminated portion visible is therefore the lune of angle BOD, and has an area proportional to the angle. This angle therefore may be employed as a measure of the phase of the planet.

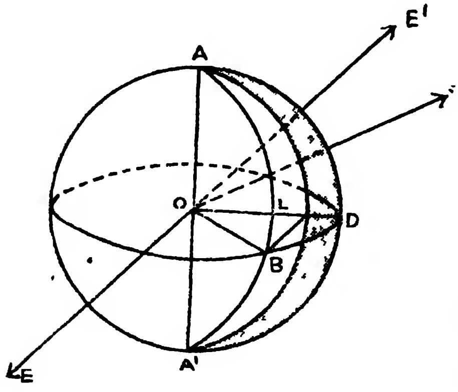

Fig 8 2

OD is perpendicular to OE′ where E′ is a point on EO produced and OB is perpendicular to OS. Hence $\angle BOD = \angle E'OS =$ supplement of $\angle EOS$.

Hence the phase of a planet is proportional to the supplement of the angle subtended by the sun and the earth at the planet.

Galileo observed Venus with his newly invented telescope and found it to be sometimes gibbous (*i.e.* more than half full) and sometimes crescent. It is interesting to notice that this was a point against the Ptolemaic theory; for according to it the angle subtended at Venus by the earth and the sun is always obtuse (see Fig. 8.1b); the supplement is therefore always acute and so the planet should be always crescent and never gibbous.

The length of the thickest part of the illuminated portion of the disc, visible to the earth, is obviously DL where BL is drawn perpendicular to OD (Fig. 8.2).

$$DL = OD - OL = OD\ (1\text{-}\cos BOD) = OD \text{ vers } BOD$$

The thickest part is therefore proportional to the versed sine of the angle supplement to that subtended by the earth and the sun at the planet

Apparent brightness of a planet.

It should be noted that the apparent brightness of a planet is not solely dependent on its phase. For example Venus, when full, is much farther off than when crescent and may actually appear fainter on account of its distance. Brightness varies inversely as the square of the distance of a luminous object. Moreover, the apparent brightness of a planet is also proportional to the area of the illuminated portion of the disc. Hence the measure of the apparent brightness of a planet

$$= \frac{c \times \text{area of the illuminated portion of the disc}}{(\text{distance})^2}\,,$$ where c is a constant depending on the reflective power of the planet and on its surface illumination

From Fig. 8.2 the brightness $B' = \dfrac{c\pi r^2(1-\cos BOD)}{2d^2}$.

where d is the distance of the planet from the observer and r is the actual radius of the disc.

8.3. Admitting that the sun is at rest and the earth is revolving round the sun (we have already shown this to be a fact in SEC. 7.2) it is not difficult to see that planets should also be revolving round him. From observation of phases of a planet over a long period, suppose it is found that it is crescent on several dates of the year. Let E_1, E_2, E_3, E_4 (Fig. 8.3a) represent the positions of the earth in its orbit on the respective dates. The planet must lie between the earth and the sun, because it is crescent. Moreover suppose that the planet is never seen in a direction diametrically opposite the sun. The obvious conclusion would be that its orbit is round the sun and inside that of the earth. These facts of observation and the conclusion arrived at apply to the planets Venus and Mercury.

Similarly suppose a planet is observed to be full and opposite the sun on various dates: and E_1, E_2, E_3, E_4 are the positions of the earth on such dates. The positions of the planet must then be represented by points such as M_1,M_2,M_3,M_4 (Fig. 8.3a). Moreover suppose the planet is never seen to be crescent. The conclusion should be that its orbit is round the sun and outside the orbit of the earth.

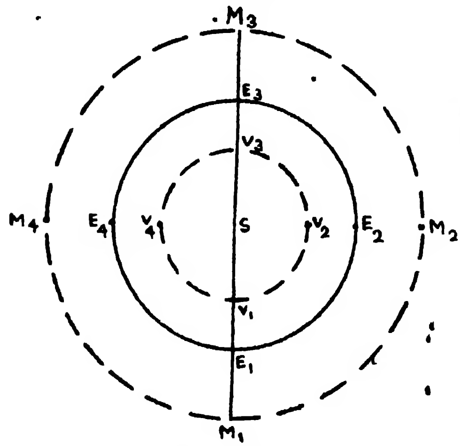

Fig. 8.3a

Definition: A planet whose orbit lies inside the orbit of the earth is called an *Inferior planet*.

Definition: A planet whose orbit lies outside the orbit of the earth is called a *Superior planet*.

Definition: When a planet is in the same direction as, and in the same line with, the sun, as seen from the earth, it is said to be in *conjunction*.

The conjunction is said to be absolute, if the planet is exactly in the same line with the sun. When however the longitude of the planet is the same as that of the sun, but they are not exactly in the same line, the planet is said to have a *conjunction in longitude*.

Definition: A conjunction is said to be *Inferior* when the planet is between the earth and the sun; and *Superior* when it is beyond the sun.

Definition: When a planet is opposite the sun and in the same straight line with him as seen from the earth, it is said to be in *Opposition*.

Opposition is said to be absolute, when the sun and the planet are exactly in the same straight line as seen from the earth. When the longitudes of the sun and the planet differ by 180° but they are not exactly in the same line, the planet is said to be in *opposition in longitude*.

In Fig. 8.3a, V_1 and V_3 are two positions respectively of inferior and superior conjunctions of the inferior planet V; M_3 is a position of superior conjunction of the superior planet M. M_1 is a position of opposition of the same planet. It is obvious that

an inferior planet cannot be in opposition, nor can a superior planet be in inferior conjunction.

Definition: The angle between the direction of the sun and that of a planet (as seen from the earth) is called its *Elongation*.

A planet is said to be in *Quadrature* when its elongation is 90°.

Definition: The interval between two successive conjunctions of the same kind or between two successive oppositions is called the *Synodic* period of the planet.

Definition: The interval between two successive passages of a planet through the same point of its orbit is called its *Periodic time* or *Sidereal period*.

It should be noted that while it is easy to observe the period between two successive conjunctions or two successive oppositions, it is impossible to observe directly the period between two successive passages of a planet through a fixed point in its orbit. For suppose a planet is at V_2 when the earth is at E_2; when the former returns to V_2 the latter will be at some other point, say E_4. In the first case the direction of the planet is E_2V_2, in the second E_4V_2, so that the planet will not be seen near the same group of stars as before and there is no observation by which we can tell directly if the planet has returned to the same point of its orbit. Sidereal period however can be calculated from the Synodic period as follows:

Observation shows that an inferior planet has a larger and a superior planet a smaller angular velocity round the sun than the earth.

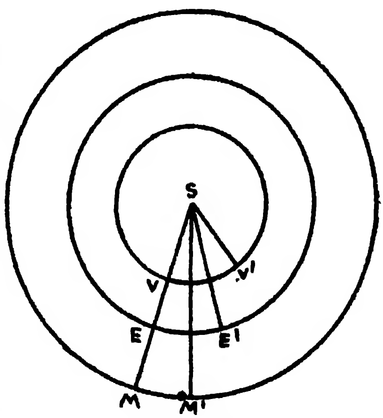

Fig. 8.3b

Let S be the sun, V an inferior planet, M a superior planet and E the earth; and let SVEM be a straight line (Fig.8.3b). Let

P = the sidereal period of V,
P′ = the sidereal period of M,
Y = the sideral period of E *i.e.*, the year,

T = the synodic period of V.
T′ = the synodic period of M.

The average angles described by V, E, M round S in a unit time (say, a day) are given by $\angle VSV' = \frac{360°}{P}$, $\angle ESE' = \frac{360°}{Y}$, $\angle MSM' = \frac{360°}{P'}$. The average

angle gained by V over E in unit time *i.e.* a day is equal to $360°/T$.

But it is also $= \angle E'SV'$. Hence,

$$\frac{360°}{T} = \frac{360°}{P} - \frac{360°}{Y}$$

Or, $$\frac{1}{T} = \frac{1}{P} - \frac{1}{Y} \quad \ldots \quad \ldots \quad \ldots \quad (8.3a)$$

Similarly the average angle gained by E over $M = \frac{360}{T'}$.

And this is also $\angle M'SE' = (360°/Y) - (360°/P')$ Hence,

$$\frac{360°}{T'} = \frac{360°}{Y} - \frac{360°}{P'}$$

or, $$\frac{1}{T'} = \frac{1}{Y} - \frac{1}{P'} \quad \ldots \quad \ldots \quad \ldots \quad (8.3b)$$

Since Y is known and T and T' can be observed, P and P' can be calculated from equations (8.3a) and (8.3b).

8.4. Tycho Brahe (Danish Astronomer 1546-1601) made the most accurate observations of his time and left a mine of data in the hands of his pupil Kepler. On the basis of these data, Kepler constructed the orbit of Mars, assuming the orbit of the earth to be a circle. This assumption did not involve him in any great error; for the eccentricity of the earth's orbit is really small. Later he considered other planets and put forward the final results in the form of the following three laws The laws go by his name and comprise the modern theory of planetary motion. Kepler's laws are:

First law Every planet moves in an elliptic orbit with the sun in one of its focuses.

Second law The areas swept out by the line joining the sun and the planet in equal times are equal.

Third law The squares of the periodic times of planets are proportional to the cubes of their mean distances from the sun

Definitions: When a body moves under a force directed towards a fixed point, it is said to describe a *Central orbit*. *An apse* of a central orbit is the point where the body is moving perpendicular to the radius vector from the centre of force. The major axis of the elliptic path of a planet is called the *line of apses* and its extremities the *apses*.

When the planet is at that extremity of the major axis of its orbit which is nearest the sun it is said to be at *perihelion;* when at the extremity farthest from the sun, it is said to be at *aphelion*.

Kepler's method of constructing the orbit of a planet is as follows:

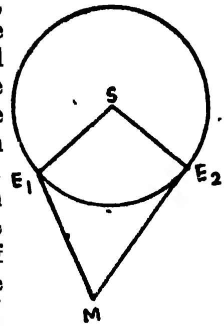

Fig. 8.4

Let S be the sun and the circle, centre S, the orbit of the earth (Fig.8.4). Suppose the elongation of a planet, say Mars, is observed on a particular day when the position of the earth is represented in the diagram by the point E_1 The direction of the planet E_1M can therefore be constructed. After a sidereal period, Mars returns to the same point M in its orbit. Let E_2 represent the position of the earth at the time. Observe the elongation of the planet and draw its direction E_2M in the diagram. The point of intersection of E_1M and E_2M is then a point on the planet's orbit. From such pairs of observations, any number of points and so the whole orbit can be obtained. At the same time the dates on which the planet is at the various points of its orbit are known; material for the construction of the first two laws is thus in hand. The third law of course follows from a comparison of different planets.

8.5. From Kepler's second law Newton showed that the force under which the planet is moving must be towards the sun; from the first, he deduced that the force varies inversely as the square of its distance from the sun; and from the third, he found that the constant of variation is the same for all planets. These results combine into his Law of Universal Gravitation which states that there is a force of attraction $= G(mm'/r^2)$ between any two particles in the universe of masses m and m', placed at a distance r from each other. G is a constant called the constant of gravitation.

It is not possible within the scope of this work, to set out Newton's deductions. But taking the planetary orbits to be circular instead of elliptical, the law of universal gravitation may be established as follows.

Let two planets P and P' revolve in circles of radii r and r' about the sun, and let T and T' be their periodic times. According to Kepler's second law, the areas swept out by the radius vector in equal times are equal, *i.e.*, equal arcs are described in equal times. It follows that the velocity in the orbit is constant in magnitude. Let the velocities of P and P' be v and v'. We know from elementary dynamics that the accelerations of P and P' are then (1) towards the centre *i.e.* the sun and (2) of magnitudes v^2/r and v'^2/r' respectively. Now from Kepler's third law we have

$$\frac{T^2}{T'^2} = \frac{r^3}{r'^3} \text{ i.e., } \frac{(2\pi r/v)^2}{(2\pi r'/v')^2} = \frac{r^3}{r'^3}$$

Or, $$\frac{v}{\sqrt{\frac{1}{r}}} = \frac{v'}{\sqrt{\frac{1}{r'}}} = \text{a constant, k, say} \quad \ldots \quad \ldots \quad (8\,5)$$

i.e. $v = k\sqrt{\frac{1}{r}}$ and $v' = k\sqrt{\frac{1}{r'}}$.

The accelerations v^2/r and v'^2/r' are therefore equal to k^2/r^2 and k^2/r'^2 respectively. That is to say the forces vary inversely as the squares of the distance between the sun and the planets.

The same inverse square law of force is found to hold in stellar systems and also among terrestrial objects. Hence the law is universal.

Conversely, assuming the law of universal gravitation, Kepler's third law can we easily deduced.

Let P and P′ be two planets revolving in circular orbits of radii r and r′ round the sun. Their accelerations according to the law of universal gravitation, are towards the sun *i.e.* towards the centre; and their ratio is $1/r^2 : 1/r'^2$. There being no acceleration in the direction of motion at any time, the velocity in the orbit remains constant in magnitude. Let the velocities be v and v′ We know from elementary dynamics that the accelerations of the planets are towards the centre and of magnitudes v^2/r and v'^2/r' Hence

$$\frac{v^2}{r} : \frac{v'^2}{r'} = \frac{1}{r^2} : \frac{1}{r'^2}$$

Since v and v′ are constants, they are equal to $(2\pi r/T)$ and $(2\pi r'/T')$ where T and T′ are the periodic times of the planets. Therefore

$$\frac{\left(\frac{2\pi r}{T}\right)^2}{r} : \frac{\left(\frac{2\pi r'}{T'}\right)^2}{r'} = \frac{1}{r^2} : \frac{1}{r'^2}$$

Or, $$\frac{T'^2 r}{T^2 r'} = \frac{r'^2}{r^2} \quad \therefore \frac{T'^2}{T^2} = \frac{r'^3}{r^3}$$

3.6. The ratio of the masses of the sun and any planet possessing a satellite may be estimated as follows:

Let S = mass of the sun

E = mass of the earth

R = the radius of the circular orbit of the earth relative to the sun.

The attraction of the sun on the earth $= G\frac{ES}{R^2}$ where G is the constant of universal gravitation. The acceleration of the earth towards the sun is therefore $= G\frac{S}{R^2}$ and the acceleration of the sun towards the earth is $= G\frac{E}{R^2}$. Hence the acceleration of the earth relative to the sun is $G\frac{E+S}{R^2}$. But since the earth is moving in a circular orbit with uniform speed v (relative to the sun), its acceleration (relative to the sun) towards the centre of the orbit is $v^2/R = \frac{4\pi^2 R}{Y^2}$ where Y is the year. Hence equating the two,

$$G.\frac{E+S}{R^2} = \frac{4\pi^2}{Y^2}R \; ; \; \text{Or} \quad E+S = \frac{4\pi^2}{G} \times \frac{R^3}{Y^2} \quad \dots \quad (8.6a)$$

Similarly, let M = mass of the moon

r = radius of its circular orbit relative to the earth

T = the sidereal period of the moon;

then as in equation (8.6a),

$$M+E = \frac{4\pi^2}{G} \times \frac{r^3}{T^2} \quad \dots \quad \dots \quad (8.6b)$$

Dividing (8.6a) by (8.6b),

$$\frac{E+S}{M+E} = \frac{R^3}{r^3} \times \frac{T^2}{Y^2} \quad \dots \quad \dots \quad (8.6c)$$

Practically E is a small fraction of S and M of E; hence without much error the equation (8.6c) can be written as

$$\frac{S}{E} = \frac{R^3}{r^3} \times \frac{T^2}{Y^2} \quad \dots \quad \dots \quad (8.6d)$$

A similar equation can be obtained with regard to any planet possessing a satellite. For instance, let J be the mass of Jupiter, R′ and r′ be the radii of its own orbit and that of a satellite, and T′ and t′ the sidereal periods of the planet and its satellite respectively. Then

$$\frac{S}{J} = \frac{R'^3}{r'^3} \times \frac{t'^2}{T'^2} \quad \dots \quad \dots \quad (8.6e)$$

Equations like (8.6d and 8.6e) yield the ratio of the masses of most planets to that of the sun. Their absolute masses could be known if the mass of any one planet be determined.

It is possible to determine the mass of the earth as follows:

Let E = the mass of the earth

a = the radius of the earth

Then the attraction of the earth on a particle on its surface, according to the law of gravitation, is $G(E/a^2)$. This acceleration, usually denoted by g, can be observed. Hence we have the relation

$$G\frac{E}{a^2}=g \qquad (8.6f)$$

The constant of universal gravitation G can be found by Cavendish's experiment, explained in books of Physics. Therefore E follows from the equation above. Hence masses of the sun and all planets possessing satellites are determined by equations (8.6d) and (8.6f).

8.7. In SEC. 8.3 and 8.4, we have developed the modern theory of planetary motion. We shall now show that it is capable of explaining all observed planetary phenomena. Strictly speaking, the trial of the theory should consist in showing agreement of all observed positions of a planet with those deduced from the theory. Within the scope of our elementary study, we have neither attempted the *complete* specification of the orbit of a planet from observational data, nor are we able to deduce its position at any time. But a qualitative explanation of a few characteristic phenomena is possible.

First we consider the question of phases of a planet. For simplicity, we take the orbit to be circular and coplanar with that of the earth.

Let the circles, with the sun S at the common centre, represent the orbits respectively of an inferior planet V, of the earth E, of a superior planet M (Fig. 8.7). Phases depend on the relative positions of the planet and the earth with respect to the sun. Let SVEM be a straight line. Consider E fixed and let V take various relative positions V_1, V_2, V_3, V_4, V_5. Let the arrowheads indicate the directions of motion of the planets in their orbits. Since the angular velocity of V round S is greater than that of E round S, the successive relative positions V_1, V_2,...... are in the direction shown by the arrow-head. (i) When the inferior planet is at V, the angle subtended at it by E and S is 180°. Its supplement is 0°. Hence no *portion of the illuminated* surface is visible (SEC. 8.2).

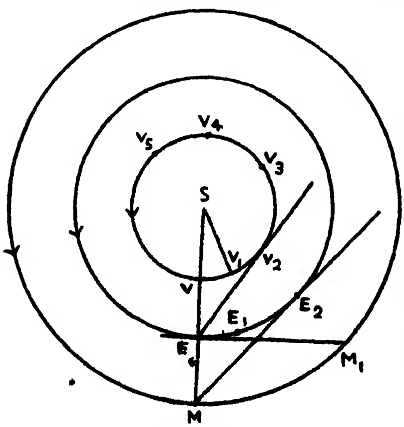

Fig. 8.7

—a fact which is also otherwise obvious. The phase of the planet is called *new*.

(ii) When the planet is at V_1 the angle subtended at it by E and S is obtuse; the supplement is acute. So less than half the illuminated surface is visible to an observer on the earth. The phase is called *crescent*.

(iii) When the planet is at V_2 where EV_2 is a tangent to the orbit, the angle subtended at it by E and S is 90°: the supplement is 90°. The planet is half full to the observer on the earth. It is then said to be *dichotomized*.

(iv) When the planet is at V_3 it is more than half full and is said to be *gibbous*.

(v) When the planet is at V_4 on ES produced, it is *full*.

(vi) When the planet is at V_5 and at points farther round, the phases repeat in the reverse order.

So an inferior planet shows all the phases from new to full. Moreover the elongation of the planet can never exceed $\angle$ SEV_2, where EV_2 is tanget to the orbit. The angle SEV_2 is obviously acute; so the planet is never very far from the sun in the sky. If it be west of the sun, it will rise before sun-rise and remain visible until the sun rises. It is then called the 'morning star'. When it is east of the sun, it will not rise before the sun, will not be visible during day-time, but will appear in the western sky after sunset. It is then called the 'evening star'.

Mercury and Venus exhibit these phenomena according to theory.

Consider now phases of a superior planet M. Since phases depend on the relative positions of the planet and the earth, with respect to the sun, we may consider the planet to be stationary at M on the outermost circle while the earth E which has a larger angular velority round S than M, takes successive relative positions on the middle circle in the direction of the arrow-head (Fig. 8.7). It is easily seen that the angle subtended at M by E and S is greatest when E is at E_2. E_2M is tangent to the orbit of the earth. In this configuration, the portion of the illuminated surface of the planet visible to the earth is the least. But $\angle$ SME_2 is necessarily acute and its supplement obtuse. Hence the least portion is still more than half the illuminated surface—a superior planet cannot be dichotomized or crescent or new. This fact is expressed by saying: '*A superior planet is most gibbous at quadrature*'.

Since the elongation SEM can have any value, the planet may, unlike an inferior planet, be visible at any time of the night. Observation agrees with these deductions from theory.

8.8. Let us now explain those special features of planetary motion described in SEC. 8.1, which gave rise to the name 'planet'.

We employ Fig. 8.7, where S represents the sun, the innermost circle the orbit of an inferior planet V, the middle circle the orbit of the earth E, and the outermost circle the orbit of a superior planet M. The arrowhead indicates the direction of motion in the respective orbits.

Consider first the case of an inferior planet V. At inferior conjunction when the planet is at V, both the earth and the planet are moving in parallel directions. From equation (8.5), we know the velocity of the planet is greater than that of the earth. Hence the relative motion of the planet is clockwise. But the relative motion of the sun is obviously counter-clockwise. Hence at inferior conjunction the motion of the planet is *retrograde*

Again consider the planet at V_2 where EV_2 is a tangent to the orbit of the planet. The planet is moving along the line of sight its own motion is therefore imperceptible to the observer on the earth. But on account of the earth's motion, the planet will appear to move counter-clockwise. The sun as before appears to move counter-clockwise. The motion of the planet is therefore *direct* in this situation.

In passing from the retrograde to the direct character of motion, the planet will obviously be *stationary* in some intermediate orientation, say E and V_1.

Consider now a superior planet. When it is in opposition at M, both the planet and the earth are moving in parallel directions; but the velocity of the earth is greater (see equation 8.5). The relative motion of the planet is therefore clockwise. Motion of the planet is thus *retrograde* at opposition

At quadrature, suppose the planet is at M_1 and the earth at E, where EM_1 is tangent to the orbit of the earth. The earth moves along the line of sight EM_1 of the planet The relative angular motion of the planet is therefore unaffected by the motion of the earth. But the planet's own motion makes it appear to move in the anti-clockwise direction. Hence the motion of the planet is, *direct*. Between direct and retrograde stages, there will be an orientation, say E_1 and M, where the motion of the planet will be stationary.

8.9. The ratio of the distance of a planet from the sun to that of the earth can be obtained from the construction of the orbit by Kepler's method. On the assumption that the two orbits are circular and coplanar, the ratio may be obtained more directly as follows:

Consider first an inferior planet. Let the circles with the sun S at the common centre represent the orbits of an inferior planet and of the earth (Fig. 8.9a). Note the time of the inferior conjunction, when the planet is at B and the earth at A After an interval t, let the relative position of the planet be B_1 the earth continuing at A; the elongation of the planet SAB_1 at the instant can be observed. Let T be the synodic period of the planet. Then $\angle ASB_1 = (2\pi/T)t$. Hence the angles of the triangle ASB_1 are known. Now,

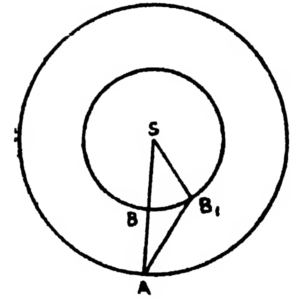

Fig. 8.9

$$\frac{SB_1}{SA} = \frac{\sin SAB_1}{\sin SB_1A} \therefore \frac{r'}{r} = \frac{\sin SAB_1}{\sin SB_1A}$$

where r' and r are the radii of the orbits of the planet and the earth respectively. The ratio of r to r' is therefore known.

The case of a superior planet can be explained with the help of the same figure Let the outer circle now represent the orbit of the planet and the inner that of the earth. Note the time of opposition when the planet is at A and the earth at B. After an interval t, let the relative positions of the earth and the planet be B_1 and A respectively The $\angle ASB_1 = (2\pi/T).t$, where T is the synodic period of the planet. The elongation of the planet SB_1A can also be observed. Thus all the angles of the triangle AB_1S are known and as before the ratio of the radii can be obtained.

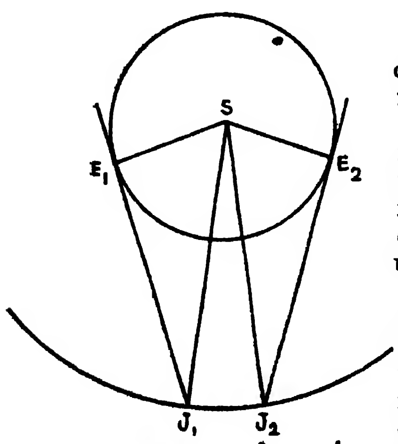

Fig. 8.9b

The ratio of the distance of the earth to the distance of the planet from the sun is called the annual *parallax* (in radians) of the planet (see SEC 14.2). On the other hand, the ratio of the radius of the earth to the distance of the planet from the centre of the earth is called the *diurnal parallax* (in radians) of the planet—(see SEC. 11.1). Diurnal parallax of a superior planet is generally very small. A method of deriving the annual parallax is the following. It should be remarked that the method is not substantially different from that given above.

In Fig 8.9b, let E_1 and E_2 be two positions of the earth in its orbit round the sun and J_1 and J_2 the corresponding positions of the superior planet at two consecutive quadratures. Let the interval between them be T. Let the known synodic period of the planet be P. Since the angle gained by the earth over the planet is $\angle E_1SJ_1 + \angle E_2SJ_2 = 2\angle E_1SJ_1$ in the time T, $2\angle E_1SJ_1 = (2\pi/P)\,T$ and is therefore known. Hence the annual parallax (in radians) $= \dfrac{E_1S}{J_1S} = \operatorname{Cos} E_1SJ_1 = \operatorname{Cos}\left(\dfrac{\pi T}{P}\right)$.

8.10. We give below the names of the planets in order of their mean distances from the sun. In the second column of the table, the mean distances are given in terms of the distance of the earth from the sun. In the third column, numbers obtained from the formula $4+3\times2^{n-1}$ for $n=1, 2, 3\ldots$ are given. The number 4 corresponds to Mercury; and $n=1, 2,\ldots\ldots$ give numbers corresponding to Venus, Earth.........

List of Planets

	Names	Mean distances from the sun	$4+3\times2^{n-1}$	
Inferior planets	Mercury	.39	4	Interior planets
	Venus	.72	7	
	Earth	1.00	10	
Superior planets	Mars ...	1.52	16	
	Asteroids ...	2.65	28	
	Jupiter ...	5.20	52	Exterior planets
	Saturn ...	9.54	100	
	Uranus ...	19.19	196	
	Neptune ...	30.07	388	
	Pluto ...	39.60	772	

There is a remarkable correspondence between the numbers in the second and the third columns, excepting in the cases of the last two planets. The formula may therefore be used to obtain the approximate distances from the sun of planets except the last two. It is known as *Bode's law*, after the name of its discoverer Bode (German Astronomer: 1747-1826). When it was first put forward, there were two prominent gaps in the series of distances: the planet Uranus and the Asteroids were still undiscovered. With the discovery of Uranus by Herschel (English Astronomer: 1738-1822) one gap was filled in agreement with the law. Later searches led to the discovery of the asteroids—a swarm of smaller masses, the distance of the centre of gravity of which from the sun was again in agreement with the law.

If we denote the distance of a planet by r, we have

$$r = 4 + 3 \times 2^{n-1} \quad = 4 + 3 \times e^{(n-1)\log 2}$$

which is of the form $r = a + b.e^{k\theta}$. The planets therefore may be supposed to have been at one time arranged on an equiangular spiral. It may be conjectured that the solar system was evolved by matter streaming out of the sun in the form of an equiangular spiral. Jeans' researches however point to a different mode of evolution of the solar system, namely tidal disruption of the sun. But Bode's law remains a mystery yet to be explained.

Examples worked out

1. If the Sun and Venus rise at the same point on the horizon, n days after the vernal equinox, find the elongation of Venus Show that $\sin(720°n/365\frac{1}{4})$ is less than 7/10. (Assume that the orbit of Venus is coincident with the ecliptic, that the rate of increase of the Sun's longitude is uniform, that the distances of the Earth and Venus from the Sun are according to Bode's law).

Since the sun and the planet rise at the same point of the horizon, their declinations are the same. The planet is of course not at the same point of the ecliptic as the sun; for then it will not be visible at all. Therefore the distance of the sun from γ is equal to the distance of the planet from ☊ measured along the ecliptic. Now the distance of the sun from $\gamma = \frac{360°n}{365\frac{1}{4}}$. $\therefore$ the elongation required $= 180° - 2\ (360°n/365\frac{1}{4}) = 180°\ (1\text{-}4n/365\frac{1}{4})$.

$\sin 180°(1\text{-}4n/365\frac{1}{4})$ must be less than the sine of the max elongation which is 7/10. $\therefore \sin 720°n/365\frac{1}{4}$ is less than 7/10.

2. Assuming that Venus and the Earth describe circular orbits in the same plane show that Venus will appear brightest at a distance ρ given by $\rho = \sqrt{b^2+3a^2} - 2b$ where a and b are the heliocentric distances of the Earth and Venus.

Let B represent the brightness of venus. Then $B = \frac{c\pi r^2}{2\rho^2}$. $(1+\cos d)$ (see SEC. 8.2) where d is the angle subtended at the planet by the earth and the sun.

$$\text{But } \cos d = \frac{\rho^2 + b^2 - a^2}{2\rho b} \quad \therefore B = \frac{c\pi r^2}{2\rho^2}\left(1 + \frac{\rho^2 + b^2 - a^2}{2\rho b}\right)$$

$$= \frac{c\pi r^2}{4b\rho^3}\ (\ \rho^2 + 2\rho b + b^2 - a^2\ )$$

For maximum brightness $\frac{dB}{d\rho} = 0$ *i.e.* $\rho^2 + 4b\rho + 3(b^2 - a^2) = 0$

$\therefore \quad \rho = (\sqrt{b^2 + 3a^2} - 2b)$, neglecting the negative root.

Exercise 8

1. Find the ratio of the masses of the sun and the earth, given that

the radius of the earth's orbit	..	93,000,000 miles
the radius of the earth ..	...	4,000 miles
acceleration due to gravity	...	32 ft./sec^2.
the year	...	$365\frac{1}{4}$ days

2. The interval between the eastern and western quadratures of Jupiter is 175 days and between two oppositions 400 days approximately. Find the distance of the planet from the sun.

3. Calculate the periodic time of a celestial body describing a circular orbit round the sun at half the distance of the earth.

4. Jupiter and Venus are evening stars, and stationary. Find which way they will begin to move.

5. Compare the velocity of Mercury with that of the earth, according to Bode's law.

6. Find the periodic time of Venus on the basis of Bode's law. Hence deduce the interval between two successive conjunctions of the same kind.

*7. If a planet rises at the east point on the 21st. June, what is its elongation? If it rises 23° 28′ North of east on the 21st. March, what is its elongation? (Consider tht orbit of the planet to be in the plane of the ecliptic).
Hint: Establish the identity of the spherical triangle with the east point, the point where the planet rises, and the First point of Aries as vertices on one hand, and the spherical traingle with the celestial pole, the zenith, and the pole of the ecliptic as vertices, on the other.

8. Compare apparent brightness of Venus when full with that when dichotomized, with respect to an observer on the earth.

9. Calculate the period during which Venus appears as a morning star at a stretch. (Assume distances of planets from the sun to be according to Bode's law).

10. Calculate the diurnal parallax of Mars at opposition, and at quadrature.
Take the radius of the earth ... 4,000 miles,
the radius of the orbit of the earth 93,000,000 miles.
ratio of the radius of the orbit of the earth to the radius of the orbit of Mars .. ... 10 :16

11. The synodic period of Mars is 780 days, and the radius of its orbit (assumed circular) is 1.52 times the radius of the earth's orbit. Calculate (*i*) its phase at quadrature, (*ii*) its elongation 195 days before opposition. (Andhra 1944).

9. COMETS AND METEORS

9.1. The motion of comets relative to stars has been briefly described in SEC. 2.1. In some respects the motion resembles that of planets. The points of difference are (1) while planets are confined within a narrow belt round the ecliptic, comets are not; (2) while all planets move round the sun in the same direction, some comets move in the opposite direction too.

Several hundreds of comets have been hitherto observed. Their orbits can be calculated, on the basis of Newton's universal law of gravitation, from *three* observations of positions separated by suitable intervals of time. It has been found that many of the orbits are parabolas or perhaps hyperbolas while the rest are probably elongated ellipses very nearly parabolic. Comets having elliptic orbits are said to be periodic, because they return to view at more or less regular intervals; those describing parabolic or hyperbolic paths do not return. It should be mentioned that both comets themselves and their paths may undergo considerable change by the gravitational influence of planets near which they may happen to pass. Planets however have been found to be hardly disturbed by the near passage of comets. This shows that masses of comets must be very small; but their volumes are generally enormous. The density of matter of comets is therefore incredibly low.

9.2. The most remarkable among periodic comets is that known as Halley's. (Halley :- Astronomer Royal, 1656—1722). From old records, Halley found that the elements of the paths of comets which appeared in 1531, 1607 and 1682 were almost identical, and concluded that is was the same comet which appeared in these years, and predicted its return in 1758. Clairaut (French Astronomer: 1713-65) introduced allowances for perturbations by Jupiter and Saturn and predicted that the comet would be closest to the sun *i.e.* at perihelion about the middle of April, 1759. Uranus and Neptune were not then discovered and no allowances were made for their perturbations. Actually the comet was at perihelion about the middle of March, 1759. The comet has since appeared in 1835 and 1910.

9.3. Meteors or shooting stars are even more transient celestial bodies—they are visible in the sky only for a few seconds. Sometimes a particular shower can be associated together by the fact that their paths, when produced, meet practically at the same point of the sky, called the *Radiant point*.

For the short time the meteors are visible their paths must be straight lines. In Fig. 9.3, let straight lines a_1b_1,

a_2b_2 etc., parallel to the diameter A_0B_0 of the celestial sphere, represent the paths of a shower of meteors. They are projected on the celestial sphere into arcs A_1B_1, A_2B_2, etc. of great circles with common diameter A_0B_0; these arcs produced backwards meet at A_0. Thus a shower with a common radian point should be regarded as moving in parallel straight lines. The direction of motion of the shower, it should be noted, is towards B_0 which is diametrically opposite to the radian point A_0. While the paths A_1B_1, A_2B_2, etc. are relative to the observer the points A_0, B_0 are independent of his positions. EB_0 therefore indicates a fixed direction.

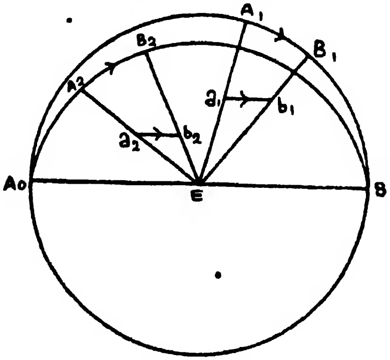

Fig. 9.3

Now suppose the path of a particular comet, no longer visible, cuts the orbit of the earth at a point and in a direction which agree with the date and the direction of the observed shower. We may then reasonably conclude that the meteors are representative of the comet—the comet has disrupted into smaller masses giving rise to the meteors These masses happen to be at the point of intersection of the orbits at the same time that the earth is there and rush into the atmosphere of the earth flashing into light due to friction of air. The shower of meteors is thus explained as the remnants of a comet.

A shower observed about the 14th November, whose radiant point is in the constellation of Leo has been associated in this way with Temple's comet. Other showers are similarly connected with other comets.

Exercise 9

1. How did Halley predict the return of the comet which goes by his name?
2. What is the radiant point? Explain that the direction of motion of a shower of meteors is towards the point diametrically opposite the radiant point
3. The radiant point of a shower of meteors is found to be 30° behind the sun on the ecliptic. Assuming the orbit of the comet which gave rise to the shower to be a parabola, in the plane of the ecliptic, find the time taken by the earth to go to the other point of intersection of the orbits.

10. THE MOON

10.1. As stated in section 2.1, the moon's path relative to stars, is a great circle. Her angular diameter varies at different points of the great circle, obviously on account of the change of her distance from the earth. Following the procedure of section 7.1, the true shape of the path is found to be an ellipse with the earth situated at a focus.

The appearance that the moon revolves in an ellipse round the earth may be produced in two ways: (1) the moon actually may revolve round the earth or (2) the earth may move round the moon. The second hypothesis however is easily shown to be wrong. It has already been shown (SEC. 7.2) that the earth moves round the sun in an ellipse with the latter situated at a focus. If it moves also round the moon, the apparent path of the moon should coincide with the ecliptic in case she is in the plane of the ecliptic, or should be a small circle (parallel to the ecliptic) in case she is out of the plane. Since the apparent path is neither the one nor the other, we conclude that the moon moves round the earth as stated in the first hypothesis.

10.2. Illumination of the surface of the moon, like those of planets, is caused by sun-rays; for otherwise the whole disc should have continuously appeared luminous. It follows that the phases of the moon depend on her positions relative to the sun and the earth as explained in SEC. 8.2.

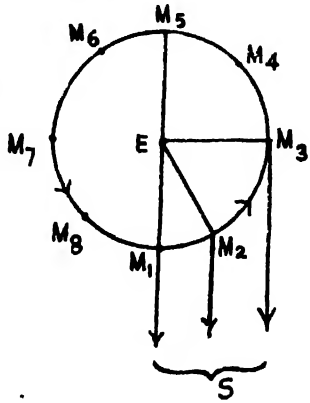

Fig. 10.2

Let E and S, Fig 10.2, represent the earth and the sun, and the circle, centre E, the orbit of the moon. The arrow-head on the circle shows the direction of motion of the moon relative to the earth and the sun. For simplicity we take the relative orbit round the earth to be a circle and in the plane of the ecliptic. The direction of the sun from any point on the circle is the same, the sun being so distant. When the moon is at M_1 on the line ES and between E and S, it is said to be in conjunction. $\angle EM_1S$ is 180° and its supplement is 0°. So no portion of the illuminated surface is visible to the earth (This of course is otherwise obvious). The phase is called *new*. At M_2 $\angle EM_2S$ is obtuse and its supplement acute: less than half the illuminated surface is

visible to the earth and the phase is said to be *crescent*. At M_3 $\angle SEM_3$, the elongation of the moon, is a right angle: the moon is at *quadrature*. The angle EM_3S is also a right angle, its supplement is 90°; and half the illuminated surface is visible. The moon is now *dichotomized*; and she is also said to be in the *first quarter*. At M_4 $\angle EM_4S$ is acute, its supplement obtuse. more than half the illuminated surface is visible to the earth and the moon is said to be *gibbous*. At M_5 on SE produced, $\angle E M_5S$ is 0°, its supplement is 180°: the whole of the illuminated surface is visible and the moon is said to be *full*. At M_6, M_7, M_8 the phases occur in the reverse order At M_1 and M_5, when the moon is in the same line with E and S, she is said to be *in syzygy*.

When the moon is crescent the dark portion of her disc is dimly visible. This is due to illumination by the earth, which also shines like the moon by the light of the sun. The phenomenon is known as *Earth-shine*. Consider the moon at M_2, in Fig. 10.2; $\angle EM_2S$ is obtuse and $\angle M_2ES$ is acute. By the principle explained in SEC. 8.2, the earth as seen from the moon, is more than half full; for the supplement of $\angle M_2ES$ is obtuse. Moreover, the reflecting power of the earth's surface (called its *albedo*) is reckoned to be about six times larger than that of the moon The earth therefore sheds strong enough light to illuminate the dark portion of the moon's disc. When the moon is more than half full the earth as seen from the moon is crescent and its light is not strong enough to make the dark portion of the moon's disc visible. The phenomenon of earth-shine is popularly described in the English language as "old moon in new moon's arms".

10.3. *Definition*: The period between two successive conjunctions or two successive oppositions of the moon is called her ***synodic period*** or the ***synodic month*** or ***lunation***.

Definition: The period between two successive passages of the moon through the same point of her orbit is called her ***sidereal period*** or the ***sidereal month***.

The synodic month or the lunation can be found out with great accuracy. From old records of eclipses, calculate the interval between the middle of a lunar eclipse of ancient times and one of recent time. The interval contains an exact number of lunations which can be easily counted. The period of a lunation is therefore obtained by division; and the result is very accurate, because even if there be an error in estimating the interval between the eclipses it is divided by a large number and becomes insignificant in the final result. The synodic month or lunation is $29\frac{1}{2}$ days or more accurately 29.5305887

mean solar days. (See SEC. 13.1 for the meaning of the term mean solar days).

The sidereal period of the moon is obtained from its synodic period in the same way as in the case of the planets (SEC. 8.3). It should however be remarked that unlike that of a planet the sidereal period of the moon may be roughly obtained by direct observation.

Let E, S, M (Fig. 10.3) represent the earth, the sun and the moon; the inner circle, centre E, is the orbit of the moon; the outer is the orbit of the sun.

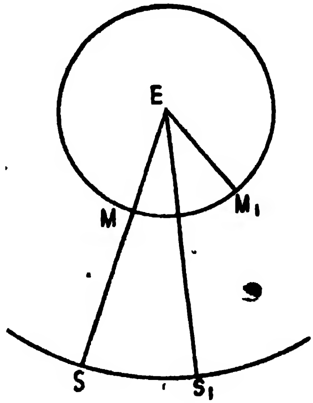

Fig 10 3

Let

S = the sidereal period of the moon,

T = the synodic period of the moon,

Y = the year *i.e.* the periodic time of the sun.

The average angle gained by the moon over the sun in a unit time = $(360°/S) - (360°/Y)$ But this angle is also $= (360°/T)$ from the definition of the synodic period. Hence

$$360°/T = 360°/S - 360°/Y$$

Or $1/T = 1/S - 1/Y$ ... (10.3)

T and Y being known, S follows. The sidereal period is found to be $27\frac{1}{3}$ days or more accurately 27.32166 mean solar days.

10.4. The motion of the moon is much complicated on account of the gravitational influence of the sun, called *perturbation;* her orbit undergoes continual changes on this account. One such change consists in the rotation of the plane of the moon's orbit about the pole of the ecliptic, that is to say, the rotation of the pole of the moon's orbit round the pole of the ecliptic. The dynamical explanation of the phenomenon is similar to that given in SEC. 14.4 in case of "Precession of Equinoxes".

Definition: The points where the moon's orbit (on the celestial sphere) cuts the ecliptic are called *nodes;* the one through which the moon crosses from the southern to the northern hemisphere is called the *ascending node*, and the other, the *descending node.*

The direction of rotation of the plane of the moon's orbit is such that the nodes move on the ecliptic in the direction opposite to that in which the sun moves; in other words, the nodes have a *retrograde* motion. The period of a complete revolution of the nodes has been found to be $18\frac{2}{3}$ years.

Let N be the period of a complete revolution of the sun with reference to a node of the moon's orbit. Then,

$$\frac{360°}{365\frac{1}{4}} + \frac{360°}{18\frac{2}{3} \times 365\frac{1}{4}} = \frac{360°}{N}$$

whence N comes out to be 346.6 days—a period which is of importance in connection with the study of eclipses.

10.5. The markings on the disc of the moon are seen to preserve practically the same positions on the disc at all times. The right conclusion to draw from this is that the moon rotates about an axis through her centre once in a period equal to the moon's sidereal period of revolution round the earth; and not that she does not rotate at all.

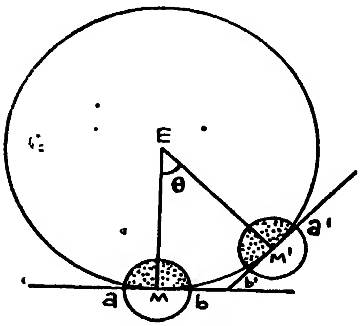

Fig. 10.5a

Consider the positions M and M′ of the moon in her orbit, separated by the radial angle θ, (Fig. 10.5a) Since the same face is turned towards the earth (the face is shaded in the figure) the bounding line ab in the position M rotates into the position a′b′ at M′ where a′b′ is the tangent to the orbit at M′. Since the angle between ab and a′b′ is easily seen to be equal to θ the period of the moon's rotation about an axis perpendicular to the plane of the orbit is equal to the period of her revolution in the orbit. Strictly speaking the angle between ab and a′b′ is not exactly θ. For while the rotation about the axis is uniform, the angular velocity of the moon round the earth is not so. The orbit of the moon round the earth is really an ellipse and the moon's angular velocity about the earth is governed by Kepler's law (SEC. 8.4). Consequently, sometimes we see a little more of the moon's surface towards the east and sometimes a little more to the west. This phenomenon is known as *Libration* in longitude

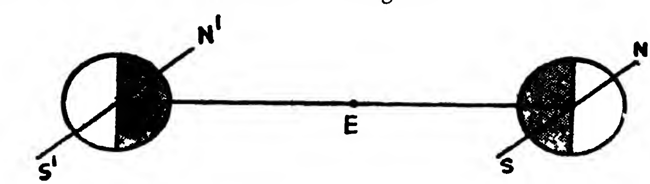

Fig. 10.5b

Again the moon's axis of rotation is not exactly perpendicular to the plane of her orbit. Consequently at one point of

her orbit, we see a little more to the south and at the diametrically opposite point, a little more to the north, as shown in Fig. 10.5b. This is known as libration in latitude. On account of the two librations, we have a view of a total of more than half—about 59%—of the surface of the moon.

10.6. Phases of the moon present a striking phenomenon in the sky and various religious rites are governed by them. Besides they are connected with tides. It is therefore important to be able to predict the phase of the moon on any date. Now it is found that $29.5306 \text{ days} \times 235 = 6939.69$ days, and

$$365.25 \quad \text{days} \times 19 = 6939.75 \text{ days}$$

The period of 19 years is therefore practically the same as that of 235 lunations and is called the *Metonic cycle* after the name of its discoverer Meton (Athenian Astronomer). The sun returns to the same point of the ecliptic and the moon occupies the same relative position with respect to the sun, after this period. The phases, which depend on the relative positions of the sun and the moon, therefore repeat in the order in which they occurred in the previous 19 years.

Therefore if we have the records of the phases of the moon in a particular year we can predict the dates of the phases 19 years after. Now this principle has been used in finding dates of full moons in any year by means of *Golden Numbers.* The golden number of a year is the remainder obtained by dividing by 19 the year of the Christian era increased by 1.

When the remainder is zero, the Golden number is 19. For instance, the Golden number for the year 1938 is the remainder obtained by dividing 1939 by 19, *i.e.* 1. The dates of full moons in 1938 are those of the year designated by the golden number 1. The dates of the full moons of a particular set of 19 successive years were inscribed on public monuments by the Athenians. The years were labelled 1, 2, 3, . . . 19, in golden letters. Hence the name 'Golden number'.

10.7. The synodic month is $29\frac{1}{2}$ days; *i.e.* the sun goes through $29\frac{1}{2}$ diurnal revolutions in this period. Since the moon falls back from the sun and loses a complete revolution in this period, the number of diurnal revolutions of the moon in $29\frac{1}{2}$ days is $28\frac{1}{2}$. The average period of a diurnal revolution of the moon is therefore

$$\frac{24 \times 29\frac{1}{2}}{28\frac{1}{2}} \text{ hours} = 24 \text{ hr. } 50 \text{ min.}$$

If the moon's path were coincident with the celestial equator, she would rise at intervals of 24 hr. 50 min.; since it is not, the interval between successive moon-rises is at times less and at times greater than the average period calculated above. The variation is the more marked in high latitudes.

Definition: The excess of the interval between successive moon-rises over 24 mean solar hours is called *Retardation.*

Retardation of the full moon at or near the autumnal equinox is particularly small and the moon is then called the *Harvest moon*, because it is supposed to help harvesters to prolong their work into night, moon-light being available shortly after sun-set.

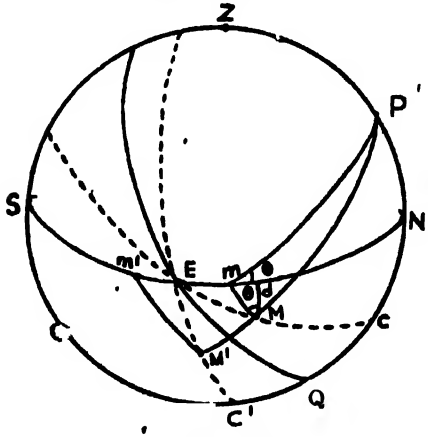

Fig 10.7

For simplicity of explanation, we consider the path of the moon to be coincident with the ecliptic At the autumnal equinox, the sun is at the First point of Libra, which at sun-set coincides with the west-point of the horizon; so the First point of Aries is at the instant coincident with the east-point of the horizon The position of the ecliptic is EC which from E eastward is north of the equator, EQ (Fig. 10.7).

The moon, if full, will be at E at sun-set on the day At the following sun-set the sun continues to be practically at the First point of Libra and the position of the ecliptic is practically the same as at the previous sun-set, but the moon will have shifted eastward to the position M. So she rises at m where the small circle Mm round the celestial pole P cuts the horizon.

Compare this state of things with that when the moon is full and at the vernal equinox. At sun-set the First point of Aries will now be at the west point and the First point of Libra at the east. The ecliptic is at the instant in the position EC′ equally inclined to the equator as EC but on the opposite side. The full moon at a vernal equinox will be at E at sun-set. At the following sun-set, the sun and the ecliptic occupy practically the same positions but the moon will have shifted eastward to M′. We may consider EM equal to EM′, each being the shift of the moon in course of a day. Also $\angle$MEQ is equal to $\angle$M′EQ, each being the obliquity of the ecliptic. It follows from the identity of the two triangles EML and EM′L that the arc MM′ is perpendicular to EQ and therefore M and M′ lie on the same hour circle PMM′. The moon at M′ at sun-set of the following day will rise at m′ where the small circle M′m′ round the celestial pole cuts the horizon.

Retardation in the first case is measured by the angle mPM and in the second by the angle m′PM′. It therefore

appears from the figure that the retardation is much smaller when the moon is full at the autumnal equinox than when she is full at the vernal equinox.

We have compared retardations of the full moon only at the two equinoxes. That on any other day can of course be found by the methods of spherical trigonometry and the harvest moon can be shown to have the least retardation. A rough solution of the problem may be as follows:

Suppose in Figure 10.7, M represents the position of the moon at sun-set following a full moon. Let d be her distance below the horizon and θ the angle the declination circle Pm makes with the horizon. The small right angled triangle with Mm as hypotenuse and d as a side may be regarded as a plane triangle The angle between d and Mm is obviously θ.

Then $Mm = \frac{d}{\cos\theta}$. $\therefore$ the retardation $= \angle MPm = \frac{d}{\cos\theta . \cos\delta}$ where δ is the declination of the moon (c.f. end of SEC. 5.10). This is least when (1) d is least (2) $\cos\theta$ is greatest, *i.e.*, θ is least and (3) $\cos\delta$ is greatest *i.e.* δ is least. Now these conditions are satisfied on the 24th September, if a full moon falls on the 23rd September. For, at sun-set on the day the ecliptic makes the smallest angle with the horizon; because the first point of Aries is at the east point and therefore K the pole of the ecliptic is on the meridian and between Z and P. Now K moves round P in a small circle and ZK is least when K is on the meridian and between Z and P. Hence d is least on the day. δ is also least on the day, because it is practically zero. And θ is also least because the declination circle passes through the east point. (See worked out Example 3, CHAPTER 2).

The full moon following the harvest moon exhibits the peculiarity of the latter in a lesser degree. It is known as the *Hunter's moon.*

10.8. Of all celestial bodies the moon is the nearest to the earth. It is therefore possible to examine her surface with great minuteness. The surface is found to abound in mountains and valleys. Heights of lunar mountains can be estimated as follows:

Let the sphere, centre O, (Fig. 10.8a) represent the moon; the directions of the sun and the earth from points on the moon are represented by arrows marked S and E respectively. Let the plane of the Circle ALB be perpendicular to the direction OE; all points on the moon's surface are seen on this plane projected in the direction EO. The semicircle ACB on the spherical surface of the moon bounds the lighted portion of the moon; the curve AHB is its projection on the plane of the

circle ALB. The shaded portion is therefore the lighted part of the moon's *disc*.

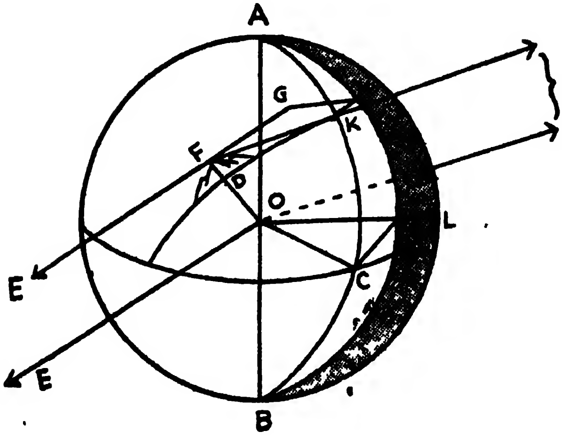

Fig. 10.8a

Now suppose there is a mountain of height DF in the prolongation of the radius OD and that its summit just catches the sun's rays. The summit will be seen as a bright spot at G, the projection of F in the direction EO on the dark part of the disc. (In the figure it is the unshaded part.)

Let FK be the tangent to the sphere parallel to the direction of the sun and let its projection on the plane of the circle ALB be GH. Obviously GH is perpendicular to the line AB joining the horns of the moon, because it lies in the plane of FE and FS, which is perpendicular to AB.

Let us measure the length GH, *i.e.* the distance of the bright spot from the boundary of the lighted part of the disc, measured in a direction perpendicular to AB. Then $FK = GH \div \sin\theta$ where $\theta = \angle SFE$ *i.e.*, the angle subtended at the moon by the earth and the sun. This angle is the supplement of the elongation of the moon.

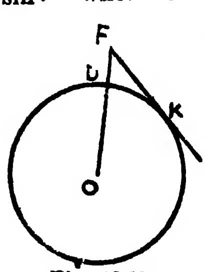

Fig. 10.8b

Now the relation between the tangent FK and the height DF (Fig. 10.8b) is given by $FD.(FD+2r) = FK^2$, where r is the radius of the moon.

Or, $FD.2r = FK^2$ approximately, since FD is small compared with r. Therefore

$$FD = \frac{GH^2}{2\,r\,\sin^2\theta} \quad (10.8)$$

Examples worked out

1. If ω and ω' be the angular velocities of the moon about the earth, and of the earth about the sun, in orbits supposed circular, and if α be the moon's greatest elongation from the earth as it might be seen from the sun, show that the times between the successive greatest elongations are alternately $\frac{\pi-2\alpha}{\omega-\omega'}$, and $\frac{\pi+2\alpha}{\omega-\omega'}$

Since the angular velocity of the earth round the sun is ω' the apparent angular velocity of the sun as seen from the earth is also $=\omega'$ in the same sense. Now the actual angular velocity of the moon $=\omega$. $\therefore$ Relative to the sun the angular velocity of the moon is $\omega-\omega'$. Hence the time to describe α (the greatest elongation) relative to the sun

$$=\left(\frac{\pi}{2}-\alpha\right)\div\left(\omega-\omega'\right)=\frac{\pi-2\alpha}{2(\omega-\omega')}$$

$\therefore$ time required to describe double the angle, *i.e.* the time from the position of one greatest elongation to the other is $=$

$\frac{2.(\pi-2\alpha)}{2(\omega-\omega')}=\frac{\pi-2\alpha}{\omega-\omega'}$. Similarly the other interval canbe found out.

2. If at a particular place the moon is in her first quarter on the 21st March, show that she will be on the meridian at 6 P.M. Show further that the time during which the moon remains above the horizon on that day is equal to the length of the mid-summer day at the place. (Assume the path of the moon to be coincident with the ecliptic and neglect her shift relative to the sun).

Since the moon is in her first quarter she will be to the east of the sun at an angular distance of 90°. The R.A. of the moon is therefore 90°. Hence when the moon is on the meridian the sun will be at the west point. But on the 21st March the sun is at the west point at 6 P.M. So the moon is on the meridian at 6 P.M. The declination of the moon is obviously 23° 28′; for the great circle joining the meridian position of the moon and the west point is the ecliptic and the angular height of the moon's meridian position above the equator is the angle between the equator and the ecliptic. $\therefore$ the diurnal circle of the moon is the same as that of the mid-summer sun. The length of time the moon is above the horizon is thus $=$ the length of the mid-summer day at the place.

Exercise 10

1. Show that if the sun were only twice as distant as the moon and the moon's synodic period were 30 days, the moon will be dichotomized only 5 days after new moon.
2. If the moon be dichotomized on the 21st March and her ascending node coincides with the first point of Aries, what will be her meridian zenith distance on the day?
3. If the reflecting power of the surface of the earth be n times that of the moon, find how many times stronger earth-light will be to an observer on the moon than moon-light to an observer on the earth (i) when either body appears full to the other, (ii) simultaneously at any particular time.
4. At a place of north latitude 45°, at what angle does the ecliptic cut the horizon at sun-set on the 23rd September? At what angle at sun set six months later?
5. Calculate the number of degrees the nodes of the moon's orbit shift backwards in a year.
6. At latitude 66°32′ north, show that the retardation of the moon, if full, will be nil on the 23rd September, assuming the moon's path to be coincident with the ecliptic.
7. Show that the height of a mountain on the surface of the moon in miles is approximately $537m^2 \operatorname{cosec}^2 e$, where m is the observed distance of the bright summit measured perpendicular to the line joining the horns in terms of the radius of the moon, and e is the moon's elongation. (Take the moon's diameter 2148 miles).
8. Find the lowest latitude at which it is possible to have a circumpolar moon.
9. If the maximum and minimum apparent diameters of the moon are 33′ 35″ and 29′ 21″ respectively find the eccentricity of the orbit of the moon.

11 DIURNAL PARALLAX

11.1 *Definition* The *diurnal* or *Geocentric parallax* of a celestial body is the angle subtended at it by the straight line joining the position of the observer to the earth's centre.

We shall take the earth to be a sphere In Fig 11 1, O is the centre of the earth, A the observer and M the celestial body. The diurnal parallax of M = $\angle OMA = p$, say. Produce OA to Z: Then $\angle ZAM = z$ is the Z D. of M Let OA, the radius of the earth, be r; and MO, the distance of M from the earth's centre be d Then from the

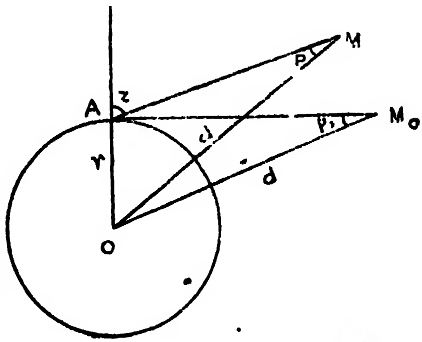

Fig 11 1

triangle OAM $\dfrac{r}{d} = \dfrac{OA}{OM} = \dfrac{\sin p}{\sin(180^\circ - z)} = \dfrac{\sin p}{\sin z}$,

Therefore $\sin p = \dfrac{r}{d} \sin z$ ... (11.1a)

In any practical case p is small, hence if it be in radians,

$$p = \frac{r}{d} \sin z \text{ approximately} \qquad (11\ 1b)$$

The maximum value of p, say p_0, is given by

$$p_0 = \frac{r}{d} \qquad (11.1c)$$

since the maximum value of sin z is 1 when z is equal to 90°. The body is then on the horizon and p_0 is called the *horizontal parallax* In terms of horizontal parallax,

$$p = p_0 \sin z \qquad (11.1d)$$

i e, the Geocentric parallax of a heavenly body varies as the sine of its apparent zenith distance.

It will be seen from equation (11 1c) that the distance of a celestial body from the centre of the earth can be found if the horizontal parallax be known; for the radius of the earth can be found by the methods of SEC. 3 2. Since $z = p + \angle ZOM$, the observed Z.D. is greater than the geo-centric Z.D. by the angle p, and the displacement takes place on the vertical through the body. On account of the similarity between the effects of Refraction and Diurnal parallax it is instructive to compare them in detail.

Points of similarity:

(1) Both refraction and diurnal parallax do not affect the azimuth of a body.

(2) Both the effects are maximum when a body is on the horizon.

Points of Difference:

(1) Diurnal parallax increases the Z.D of a body but refraction decreases it.

(2) Diurnal parallax of a body depends on its distance from the earth, while refraction is independent of it

(3) The equation $p = p_0 \sin z$ is true for all Z.D.'s but $r = k \tan z$ is true for Z.D.'s upto about 70°.

11.2. The *horizontal parallax* (and hence the distance from the earth's centre) *of the moon*, our nearest neighbour, can be found as follows: ~~Bessel's method~~

Let the circle, centre O, (Fig. 11.2) represent a meridian of the earth, A and B be two observers stationed on the meridian and M the position of the moon when in the plane AOB *i.e.*, on the common celestial meridian of both the observers. Let

z = Z.D. and p = the parallax of M at A,
z' = Z.D. and p' = the parallax of M at B,
and p_0 = the horizontal parallax of M.
Then $p = p_0 \sin z$ and $p' = p_0 \sin z'$
Therefore $p + p' = p_0 (\sin z + \sin z')$

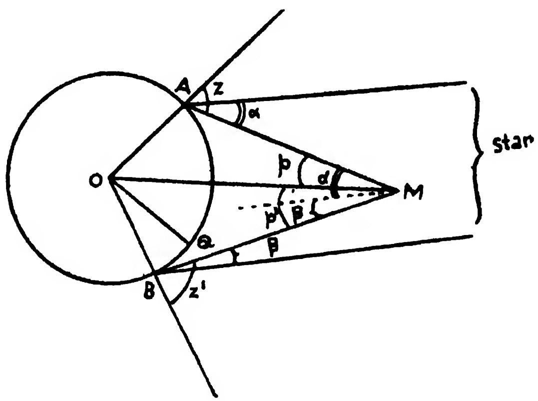

Fig. 11.2

$$\text{and } p_0 = \frac{p+p'}{\sin z + \sin z'} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11.2a)$$

z and z' can be observed; p_0 will therefore be known if we can find $p + p'$.

In the quadrilateral OAMB, $\angle$ OAM and $\angle$OBM are 180°-z and 180°-z′ respectively. Let the latitudes, north and south, of A and B respectively be φ and φ'. Then $\angle$ AOB $=\varphi+\varphi'$. The remaining angle of the quadrilateral $\angle$ AMB is therefore

$$=360^\circ-(180^\circ-z+180^\circ-z'+\varphi+\varphi')$$

i.e. $p+p'=z+z'-\varphi-\varphi'$

$$\therefore\ p_0=\frac{z+z'-\varphi-\varphi'}{\sin z+\sin z'} \qquad \ldots \qquad \ldots \qquad (11.2b)$$

substituting for $p+p'$ in equation (11.2a).

An alternative procedure to get $p+p'$ which does not necessitate accurate determination of the latitudes and the Z.D.'s is the following. Observe the angular distances of the moon from a very close star on the meridian from the two stations A and B; let them be α and β The star being infinitely distant, its directions from A and B are parallel. Draw a third straight line parallel to them through M It easily follows that in situation represented in the figure, $p+p'=\alpha+\beta$. Hence

$$p_0=\frac{\alpha+\beta}{\sin z+\sin z'} \qquad \ldots \qquad \ldots \qquad (11.2c)$$

To estimate the effect on p_0, of the errors of measurement of α, β, z, z′, differentiate the equation (11.2c);

$$dp_0=\frac{d\alpha}{\sin z+\sin z'}+\frac{d\beta}{\sin z+\sin z'}-\frac{(\alpha+\beta)\cos z\,dz}{(\sin z+\sin z')^2}$$
$$-\frac{(\alpha+\beta)\cos z'\,dz'}{(\sin z+\sin z')^2}$$

$d\alpha$, $d\beta$, dz, dz′, may be looked upon as the errors of measurement. The effect of $d\alpha$ and $d\beta$ on dp_0 will be minimized when the denominator $\sin z+\sin z'$ is as large as possible *i.e.*, when z and z′ are both as nearly 90° as possible. The same condition minimizes the effect of the errors dz and dz′ in a double way; for the condition makes not only the denominators large but also the numerators small. The latter are moreover multiplied by the small quantity $(\alpha+\beta)$. Hence errors in z and z′, under this condition, have little effect on the value of p_0. In other words, the best results are obtained by stationing the observers as far north and south respectively as possible.

Instead of stationing two observers at two different places, the moon may be observed at rising and setting by the same observer; the diurnal displacement of the observer will then form the base line for calculation of the moon's parallax.

The average horizontal parallax of the moon is 57′ 2″.

11.3. The sun's horizontal parallax is a fundamental quantity in Astronomy; for the sun's distance is the basis of calculation of astronomical distances. The method of the pre-

vious section however is not available for determination of solar parallax. Equation (11.2c) obviously cannot be employed as no star in the vicinity of the sun is visible. Equation (11.2b) involves quantities errors of whose measurement may easily be too large compared with solar parallax itself. But indirect geometrical methods are available for the purpose besides methods based on dynamical and physical considerations. First we consider geometrical methods.

(1) *Solar parallax by the parallax of Mars in opposition*:

The orbits of the earth and planets can be mapped to scale by Kepler's method. The actual distances in miles can therefore be found when one of the distances on the map is known.

Suppose S, E, M, are the sun, the earth and Mars in opposition (Fig. 11.3a). Mars being now quite close to the earth, its diurnal parallax becomes appreciable and can be obtained by the method of the previous section. The distance EM therefore becomes known. Let ES = r and MS = r′; then r/r′ is known, since orbits of the earth and Mars can be drawn to scale. Also r′ − r (*i.e.* EM) becomes known from the observed parallax of Mars. Hence r, the distance of the sun from the earth, can be solved from the two equations.

Fig. 11.3a

Some of the asteroids which come closer to the earth at opposition may be employed with greater advantage.

(2) *Solar parallax by the transit of Venus*:

The orbit of Venus is inclined to the orbit of the earth. Hence the sun, Venus and the earth are not exactly in a straight line at every *inferior* conjunction. But when the conjunction takes place at or near a node (see definition in SEC. 10.4) the three bodies are practically in a straight line. Consequently Venus is seen projected on the disc of the sun as a dark spot, moving across on account of Venus' relative motion. The phenomenon is known as the transit of Venus.

The periodic time of Venus is 224.7 days. Now,

$$8 \times 365.24 = 2922 \text{ nearly; also,}$$
$$13 \times 224.7 = 2921 \text{ nearly.}$$

Hence if a transit of Venus takes place at or near a node, another is likely to take place 8 years after; for both the earth and Venus will return to the same point (*i.e.* a point near the node) of their respective orbits. A *third* transit will not, however occur after another 8 years; on account of the difference of a day between 8 years and 13 complete revolutions of Venus, the conjunction will happen too far away from the node. But,

$$235 \times 365.24 = 85835 \text{ nearly, and}$$
$$382 \times 224.7 = 85835 \text{ nearly.}$$

Hence transit at a node will recur at intervals of 235 years. These rare occasions have been availed of for observations leading to solar parallax. But it is now held that the accuracy of the result is not as high as with other methods.

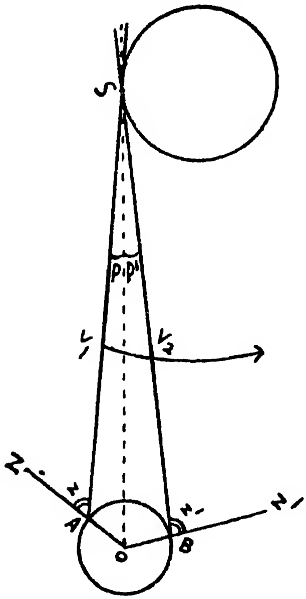

Fig. 11.3b

(a) *Delisle's method*:

For simplicity of explanation, we shall consider the orbit of Venus to be in the plane of the earth's equator. The earth's equator is represented by the circle, centre O (Fig. 11.3b). Station two observers A and B on the equator as far apart as possible. Let each note the *instant of internal contact* of Venus with the sun's disc, the respective positions of Venus being V_1 and V_2 relative to the sun and the earth Then AV_1S and BV_2S are tangents to the sun: V_1V_2 is the arc described by Venus relative to the earth in the interval between the two contacts. Since the synodic period of Venus is known, the angle ASB, gained by Venus over the earth in the interval, can be calculated. This angle, it will be noticed, is the sum of the parallaxes of the sun from A and B. Thus $p+p'$ of equation (11.2a) is known and z and z' the zenith distances of the sun at A and B at the two instants can be observed. Hence the parallax of the sun is obtained from equation (11.2a).

It should be remarked that A and B are not simultaneous positions of the observers, but positions they occupy at the instants of observation. Allowance on account of diurnal motion of the earth should therefore be made in the calculation.

(b) *Halley's method*:

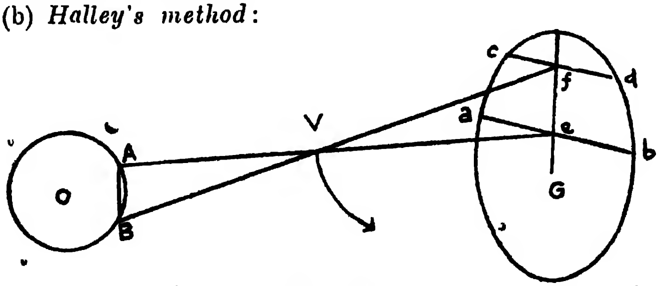

Fig. 11.3c

In Fig. 11.3c, the circle, centre O, represents a meridian of the earth; the oval curve cdba represents the disc of the

sun perpendicular to the plane of the paper; and A and B are two observers on the meridian as far apart as possible. V is Venus and the curve through V represents a portion of its path whose plane is nearly perpendicular to the plane of the paper.

As it moves, Venus is seen by the observers A and B as a dark spot describing the chords ab and cd respectively on the disc of the sun Each observer notes the *duration* of transit from which the angular values of the chords are obtained as follows:

Let S and E (Fig. 11 3d) represent the sun and the earth. Relatively to them Venus V moves in the direction VV′. Let SVE be a straight line. The rate of description of the angle VSV′ is $2\pi/T$ per unit time, where T is the synodic period of Venus Let $\angle SEV' = \varphi$ and $\angle VSV' = \theta$, where V and V′ are the relative positions at the beginning and end of a small interval

Fig. 11 3d

From the triangle SEV′, $\dfrac{\sin\theta}{\sin\varphi} = \dfrac{EV'}{SV'}$

or assuming the angles to be small $\dfrac{\theta}{\varphi} = \dfrac{r'-r}{r}$

where r is the distance of Venus from the sun = 7 units say; and r′ is the distance of the earth from the sun = 10 units approximately. Therefore, $\dfrac{d\varphi}{dt} = \dfrac{r}{r'-r} \times \dfrac{d\theta}{dt} = \dfrac{7}{3} \times \dfrac{2\pi}{T}$

$= 4''$ per minute approximately .. (11.3a)

(T = 584 days)

As one minute of time can be observed with a smaller percentage of error than four seconds of arc, the measures of the chords are more reliably obtained by noting the durations of transit.

Suppose gef (Fig. 11.3c) is the radius of the sun perpendicular to the parallel chords ab and cd seen to be described by the dark spot from A and B. The measure of the arc ef is easily obtained from the right angled triangles gbe and gdf The measure of ef in miles is obtained from the similar triangles ABV and efV. We have

$$\frac{ef}{AB} = \frac{eV}{AV} = \frac{7}{3}$$

Therefore, ef = (7/3) AB ... ... (11.3b)

AB in miles can be calculated from the positions of the observers. Hence ef in miles is found from equation (11.3b). Also the angle subtended by ef at the earth has been calculated. The distance and so the parallax of the sun therefore easily follow.

There are comparative advantages and disadvantages of both Delisle's and Halley's methods. Since they are inferior to other methods, we shall not enter into a discussion about them.

(3) *Dynamical methods* of obtaining solar parallax make use of perturbations of the moon by the sun or of Venus by the earth The explanations are beyond our scope. It should however be remarked that they are capable of yielding very accurate results. Le Verrier (French astronomer : 1811-77) who developed the method was so sure of it that he took no interest in the transit observations of Venus of his time.

(4) *Solar Parallax from the constant of Aberration*:

We have made a passing reference to aberration of stars in SEC. 7.2. It is shown in SEC 14.1 that if α be the maximum displacement of a star due to aberration, measured in radians,

$$\alpha = v/V \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (11.3c)$$

where v is the velocity of the earth in its orbit, and V is the velocity of light.

The maximum displacement of a star due to aberration, can be observed and V has been measured by laboratory methods. v therefore follows. Since the period of a complete revolution of the earth round the sun, namely the year, is known, the circumference of the orbit and hence its radius is obtained.

(5) *Solar Parallax by the Spectroscopic method*:

It has been mentioned in SEC. 7 2 that dark lines in the spectrum of a star situated along the direction of motion of the earth shift towards the violet end on account of Doppler effect The measurement of the shift yields the volocity of the earth with accuracy. Hence the circumference and so the radius of of the earth's orbit are obtained.

Different methods give results in fair agreement with one another; solar parallax is now considered to be 8″ 8. The distance of the sun from the earth is therefore known; and its radius in miles, volume, mass and average density can be deduced.

Example Worked out

If the observed Z.D's, z_1 and z_2, of two celestial bodies be also their true geo-centric Z.D.'s show that

$p_1 \cos z_1 = p_2 \cos z_2$, where p_1 and p_2 are the horizontal parallaxes of the bodies.

Let O be the centre of the earth, A be an observer and M the true position of a body (Fig. 11.1). Let M be seen in the direction AM′ (M′ is not shown in the figure) so that $\angle M'AZ = z_1$. By the condition of the problem $z_1 = \angle AOM$

$$\therefore \text{ AM}' \text{ is parallel to OM}$$

Hence $\angle M'AM = \angle OMA$

i.e., $r \tan z_1 = p_1 \sin z_1$. (Since $\angle$ M′AM and $\angle$OMA are respectively the corrections for refraction and parallax of the body.)

i.e., $r = p_1 \cos z_1$.

Similarly for the other body

$$r = p_2 \cos z_2$$

Hence

$$p_1 \cos z_1 = p_2 \cos z_2.$$

Exercise 11

1. The moon's horizontal parallax is 57′. What is the angular diameter of the earth as seen from the moon?
2. The moon's horizontal parallax is 57′ and its angular diameter 32′. Compare the radii of the moon and the earth.
3. Find the distance of the sun, given that the coefficient of aberration (*i.e.* maximum aberration) is 20″.5 and the velocity of light is 1,86,000 miles per second. (Log table may be used.)
4. The sun's parallax is 8″.2 and his angular diameter 32′. Find his diameter in miles. (Take the radius of the earth to be 4,000 miles.)
5. If p″ be the horizontal parallax of a celestial body and k″ be the coefficient of refraction, what observed zenith distance of the body will not need any correction for refraction and for reduction to the centre of the earth?
6. An observer on the equator of the earth finds the diurnal path of the moon to be the prime vertical. If the angular distance of the moon's centre from a neighbouring star be a (west) at rising and b (east) at setting, and if the moon moves through an angle c in the mean time, show that her horizontal parallax $= (a+b+c)/2$.
7. Calculate the duration of the transit of Venus if central. (Take distances according to Bode's law.)
8. Calculate the ratio of the average density of the sun to that of the earth. (See Ex. 1, Exercise 8. Base your calculation on the result of the example. Log. table may be used).
9. Assuming the earth to be spherical, show that parallax increases the apparent semi-diameter of the moon in the ratio sin z : sin(z-p), where z is the apparent zenith distance of the moon's centre and p is the angle subtended at the moon by the observer and the earth's centre.

12. ECLIPSES

12.1. Lunar eclipse is caused by the shadow of the earth cast by the sun, falling upon the moon's disc. Since the moon shines by sun-light the portion of the disc on which the shadow falls does not shine and is therefore obscured to *all* observers. The same phase of the eclipse is visible to all observers on the earth.

Solar eclipse is caused by the moon intervening between the observer and the sun. The sun is self-luminous; the portion of his disc which will be hidden by the moon depends on the position of the observer. The phase observed of a solar eclipse is thus different at different stations of the earth; a solar eclipse may be visible at one point of the earth but not at another; it may be full at once place but only partial at another.

If the orbit of the moon were in the plane of the ecliptic, there would have been a solar eclipse at every new moon and a lunar at every full moon; for then the sun, the moon, and the earth would be in a straight line at both the new and the full moon; the moon would be between the earth and the sun in the first case, and the earth between the sun and the moon in the second case. The moon's orbit however is inclined to the ecliptic and so ordinarily the three bodies are not in a straight line at new or full moons *i.e.*, when the longitudes of the sun and the moon are either equal or differ by 180°. So an eclipse does not take place at every new or full moon.

In the next section, we investigate the conditions for a lunar eclipse and further on those for a solar eclipse.

12.2. Let the circles, with centres S, E and M, represent the sun, the earth and the moon (Fig. 12.2). Draw direct

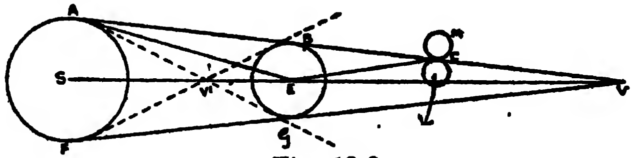

Fig. 12.2

common tangents to the earth and the sun, to form a cone with vertex V. The portion of the cone beyond the earth is called *umbra*; the sun is completely invisible from any point in it. Similarly, the portion of the cone formed by transverse tangents (shown by dotted lines in the figure) beyond the earth and not included in the umbra is called *penumbra*; part only of the sun is visible from any point in it.

When the moon enters the penumbra, there is a diminution of brightness hardly perceptible; but when she enters the umbra, its contour is perceptible on her disc and a lunar eclipse is said to happen. The contour, when seen through a telescope, is not however sharply defined on account of penetration of light by refraction through the earth's atmosphere.

The condition for the beginning of a partial lunar eclipse is that the moon should touch the umbral cone externally. Condition for the beginning of a total lunar eclipse is that she should touch the cone internally.

Let A, B, C be the points of contact of the umbral cone with the sun, the earth, and the moon respectively. Join EA, EC, EV. In the triangle AEV,

the exterior angle $AES = r_s$, the angular radius of the sun;
the interior angle $EAV = p_s$, the horizontal parallax of the sun, since it is subtended at the sun by a radius of the earth.

Therefore the angle $BVE = r_s - p_s$.

Again in the triangle EVC,

the exterior angle $BCE = p_m$, the horizontal parallax of the moon, being subtended at the moon by a radius of the earth.

Therefore, the angular radius of the shadow from the earth's centre

$$= \angle CEV = \angle BCE - \angle BVE$$
$$= p_m - (r_s - p_s) = p_m + p_s - r_s \quad \dots \quad (12.2a)$$

From practical observation it has been found that the shadow should be considered wider by about 2 per cent on account of the opaqueness of the earth's atmosphere. Taking this correction into account, the condition for a partial lunar eclipse is that the angular distance between the centres of the moon and the shadow should be equal to or less than

$$\tfrac{51}{50}(p_m + p_s - r_s) + r_m \quad \dots \quad \dots \quad (12.2b)$$

where r_m is the angular radius of the moon; the condition for a total lunar eclipse is that the distance between the centres should be equal to or less than

$$\tfrac{51}{50}(p_m + p_s - r_s) - r_m \quad \dots \quad \dots \quad (12.2c)$$

Taking $p_m = 57'$, $p_s = 9''$ and $r_s = 16'$ the expression $\tfrac{51}{50}(p_m + p_s - r_s)$ is very nearly $42'$. This indicates (eqn. 12.2a) the angular radius of the shadow at the moon's distance.

Definition: The greatest distance of the centre of the earth's shadow from the node of the moon's orbit (which is the same as the distance of the centre of the sun from the opposite node), so that a lunar eclipse may happen is called the *Lunar ecliptic limit.*

The maximum angular distance between the centres of the shadow and the moon, so that a lunar eclipse may happen has been found above. The inclination of the orbit of the moon to the ecliptic as well as the rates of motion of the sun and the moon in their paths is known. From these the lunar ecliptic limit defined above can be calculated. As the various quantities involved are not constants, there are a maxi-

mum and a minimum value of the limit. The maximum value is called the *Major* ecliptic limit, the minimum the *Minor* ecliptic limit. They are 12° 15′ and 9° 30′ respectively.

12.3. From the table of position of the sun, if he is found, at a full moon, to be within the major ecliptic limit, there is *probability*, and if within the minor ecliptic limit, *certainty*, of a lunar eclipse.

Calculation of a lunar eclipse may be carried out as follows:

Draw a straight line $Vm_1m_0Sm_2$ on a graph paper to represent the ecliptic; and let S and M be the centres of the shadow and the moon at conjunction; MS is perpendicular to the ecliptic and equal to the latitude of the moon at conjunction. Now construct the path of the moon *relative to the shadow* thus: Take V on the ecliptic such that SV : SM = the rate of the relative motion of the moon in longitude : the rate of her relative motion in latitude.

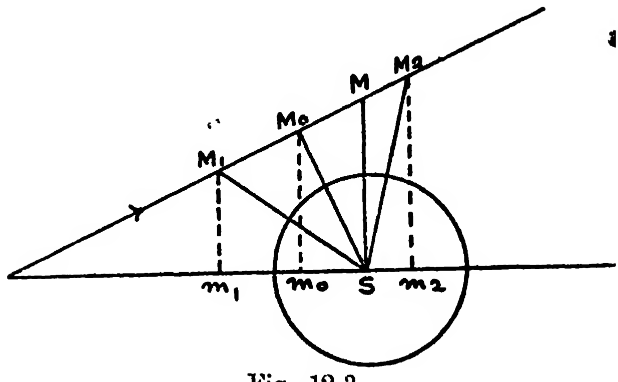

Fig. 12.3

Let the arrow-head mark the direction of the motion of the moon relative to the shadow. Note the time of conjunction at S. Since the rate of relative motion of the moon in longitude is known, a scale of time can be set down on the straight line VS.

Now with centre S and radius equal to the sum of the radii of the shadow and the moon draw an arc to cut VM at M_1 and M_2. These are the positions of the moon on her relative path at the beginning and the end of the eclipse. The corresponding times can be read at their projections m_1 and m_2 on the straight line VS, from the scale already laid down. Draw SM_0 perpendicular to VM; then M_0 is the position of the moon on the relative path at the middle of the eclipse; the corresponding time is read at its projection m_0 on the scale.

Other details, for example, the times of beginning and end of totality of the eclipse, or if the eclipse be partial, the extent of the eclipse, can be similarly calculated.

12.4. Calculation of solar eclipse is complicated, on account of the fact that the same aspect of the eclipse is not observed at all places. We shall content ourselves with the calculation of the angle between the centres of the sun and the moon, as would be observed from the earth's centre, so that an eclipse may happen at *some* place of the earth.

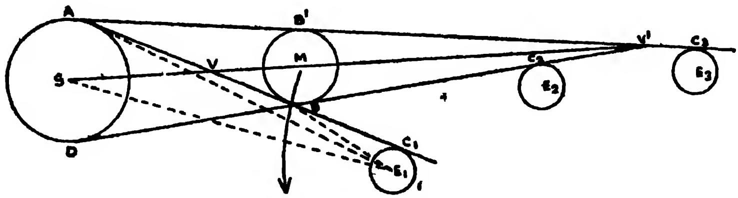

Fig. 12.4

Let the circles, with centres S and M, represent the sun and the moon; and the circles with centres E_1, E_2, E_3, different positions of the earth (Fig. 12.4). The arrow-head shows the direction of motion of the moon relative to the sun and the earth. To consider conditions for partial solar eclipses, construct the penumbral cone of shadow by drawing transverse tangents to the sun and the moon. Let the earth in the position E_1 touch the cone at C_1 so that the moon is between the earth and the sun. A partial solar eclipse is just to happen at the point C_1 of the earth's surface; for the discs of the sun and the moon are just in external contact as seen from the point C_1. Join E_1S, E_1A, E_1B. Consider the traiangle AE_1B;

$$\angle AE_1B = \angle C_1BE_1 - \angle C_1AE_1$$

$= p_m - p_s$, where p_m and p_s are the parallaxes of the moon and the sun respectively.

$\angle AE_1S = r_s$, the angular radius of the sun, as seen from the earth's centre.

Hence, $\angle SE_1B = p_m - p_s + r_s$.

So when a *partial* solar eclipse is about to happen at *some* place on the earth's surface, the distance between the centres of the sun and the moon (as seen from the earth's centre)

$$= p_m - p_s + r_s + r_m \quad \ldots \quad \ldots \quad (12.4a)$$

where r_m is the angular radius of the moon (as seen from the earth's centre).

To consider conditions for total or annular solar eclipse, construct the umbral cone of shadow of the moon, of which let V′ be the vertex. The relative distances of the three bodies vary in such a way that the earth may touch the cone at times between the moon and the vertex V′ and at times beyond V′. From any point of the umbral cone, between the vertex and the moon, the sun will be found completely hidden; while from a point on and inside the cone and beyond the vertex, only the central portion is observed hidden. This can be verified by drawing tangents to the moon from a point in the umbra between V′ and M and from a point beyond V′.

Suppose the earth in the position E_2 touches the umbral cone at the point C_2 between V′ and M. Join DE_2 and BE_2.

$$\angle DE_2B = \angle C_2BE_2 - \angle C_2DE_2 = p_m - p_s$$

Hence the angle between the centres of the sun and the moon (as seen from the earth's centre) so that a *total* solar eclipse may happen at *some* place on the earth is equal to

$$p_m - p_s + r_m - r_s \quad \dots \quad \dots \quad \dots \quad (12.4b)$$

Suppose the earth in the position E_3 touches the umbral cone at C_3 beyond V′; so that $AB'C_3$ is a common tangent to the sun, the moon and the earth, touching them at A, B′ and C_3 respectively. Join AE_3 and $B'E_3$.

$$\angle AE_3B' = \angle C_3B'E_3 - \angle C_3AE_3 = p_m - p_s$$

The angle between the centres of the sun and the moon (as seen from the earth's centre) so that an *annular* solar eclipse may happen at *some* place on the earth is

$$p_m - p_s + r_s - r_m \quad \dots \quad \dots \quad \dots \quad (12.4c)$$

Definition: The maximum distance of the sun's centre from the node of the moon's orbit, so that a (partial) solar eclipse can just take place at some position on the earth's surface, is called the *solar ecliptic limit.*

The maximum value possible of the limit is called the *major* and the minimum value possible, the *minor*, solar ecliptic limits. At a new moon, if the sun be within the major solar ecliptic limit, there may or may not be any solar eclipse; but if the sun be within the minor solar ecliptic limit, an eclipse must occur at some place on the earth; of course it will not be visible at all places.

The major and the minor solar ecliptic limits can be calculated, as in the case of the lunar limits, by taking suitable values of the constants. They are found to be 18°½ and 15°½ respectively.

12.5. The maximum and minimum numbers of eclipses possible in a year can be calculated as follows The necessary data are the following.

Lunar ecliptic limits: major 12°, minor 9°½ (SEC. 12.2)
Solar ecliptic limits. major 18°½, minor 15°½ (SEC. 12 4)
Arc described by the sun on the ecliptic relative to the moon's node, in a lunation $= \frac{360° \times 29\frac{1}{2}}{346\ 6} = 30°\frac{2}{3}$ nearly (since the period of a synodic revolution of the node is 346.6 days approximately).

Time the sun takes to go from one node to another
$= 173$ days ... (SEC. 10.2)
Time of six lunations $= 29\frac{1}{2} \times 6$ days
$= 177$ days ... (SEC. 10 2)

Let the circle (Fig. 12.5) represent the ecliptic. N,n are the nodes, L,L′ and l,l′ mark the lunar ecliptic limits on either side of the nodes. Similarly S,S′ and s,s′ mark the solar ecliptic limits.

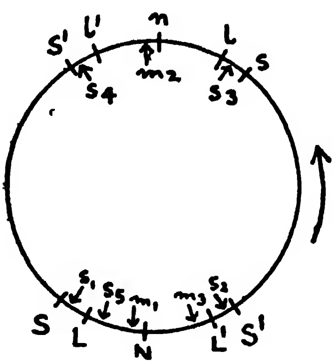

Fig 12.5

To find *the least number of eclipses possible in a year*, minor ecliptic limits should be taken.

The arc described by the sun relative to the node in a lunation is 30°⅔. But LL′$=ll'=19°$. Hence there may not be a full moon during the passage of the sun through LL′ or ll'; no lunar eclipse may therefore happen. But SS′$=$ss′$=31°$. So there will be at least one new moon during the passage of the sun through SS′ or ss′. Hence a solar eclipse must occur during the passage of the sun through them.

The least number of eclipses in a year is thus two, both of the sun.

To find *the greatest number of eclipses in a year*, major ecliptic limits must be taken. So now LL′$=ll'=24°$; SS′$=$ss′$=37°$. To secure the occurrence of the maximum number of eclipses while the sun describes SS′ suppose there is a full moon, when the sun is at m_1, *two days before* he reaches the node N. A lunar eclipse takes place, for obviously the sun is

within the lunar ecliptic limits. Half a lunation earlier there will be a new moon and the sun will be at s_1 such that $s_1m_1 = 15°\frac{1}{4}$. Nm_1 is about 2°; for the sun describes about 1° with respect to the node in a day (30°$\frac{2}{3}$ in 29½ days). So $Ns_1 = 17°$ nearly which is well within the solar ecliptic limit. There will therefore be a solar eclipse. Half a lunation after the sun has been at m_1, he will be at s_2 where $Ns_2 = 15° - 2° = 13°$ nearly, and another solar eclipse takes place.

Six lunations after the sun has been at m_1, he will be at m_2 having crossed the opposite node n two days earlier; there will now be a full moon and a lunar eclipse takes place. From considerations similar to those given previously, we should find that there will be solar eclipses half a lunation earlier and half a lunation later when the sun is at s_3 and s_4 respectively.

Six lunations after the sun has been at m_2, he will be at m_3, having passed the node N six days earlier. Nm_3 is therefore about 6° and therefore there will be a lunar eclipse. Half a lunation earlier from the position m_3, the sun is at s_5; the moon is new and the sun is well within the solar ecliptic limit; there is therefore a solar eclipse.

Starting from the position s_1 of the sun and ending at the position m_3, the interval is 12½ lunations *i.e.* 368 days; and we have counted 8 eclipses, 5 of the sun and 3 of the moon. The period is a little over one year; so in counting the number of eclipses in a year we must exclude either the first which is solar or the last which is lunar.

Thus the greatest number of eclipses in a year is seven; either 5 solar and 2 lunar, or 4 solar and 3 lunar.

12.6. We conclude our account of eclipses with the explanation of an interesting period known as the Saros discovered in pre-historic time by the Chaldeans.

$$19 \times 346.6 \text{ days} = 6585 \text{ days nearly}$$
$$223 \times 29.5 \text{ days} = 6585 \text{ days nearly.}$$

So 19 revolutions of the sun with respect to the moon's nodes occupy the same period as 223 lunations. The period is 18 years 11 days if there be four leap years or 18 years 10 days if there be five. After this period, the sun and the moon return to the same positions with respect to the nodes; and consequently eclipses recur in the same order. On account however of a small difference between the two periods the character of the corresponding eclipses, particularly solar, at the same place may not be exactly the same.

Example Worked Out

Show that the maximum duration of a lunar eclipse is roughly 3 hr. 42 m. Show also that the longest interval during which a lunar eclipse may be total is about 1 hr. 48 m. (Take $p_m=57'$, $p_s=9''$, $r_m=15'$ and $r_s=16'$).

From equation (12.2b) we find that for a lunar eclipse the angle between the centres of the moon and the shadow is less than or equal to 57′. Hence the time taken by the moon to describe 114′ ($=2\times 57'$) will give the maximum duration of a lunar eclipse. Now relative to the direction of the sun the moon describes 360° in $29\frac{1}{2}$ days. Therefore the time taken by her to describe $114'=\frac{29\frac{1}{2}}{360}\times\frac{114}{60}$ days $=3$ hr. 42 m. nearly.

For total lunar eclipse angle between the centres of the moon and the shadow is less than or equal to 27′ (Eqn. 12.2c). Therefore the time taken by the moon to describe 54′ ($=2\times 27'$) gives the maximum duration of a total lunar eclipse. But the time required to describe $54'=\frac{29\frac{1}{2}}{360}\times\frac{54}{60}$ days $=1$ hr. 48 m. nearly.

Exercise 12

1 Find the moon's greatest distance in miles from the plane of the ecliptic when she is 240,000 miles from the earth.
(Take the inclination of the orbit of the moon to the ecliptic to be 5° and $\pi=\frac{22}{7}$)

2. Find the speed of the moon in her orbit in miles per second. Supposing the moon to be at the zenith of an observer at the equator, what is her speed relative to the earth in miles per second? (Distance of the moon $=60\times 4000$ miles. Her sidereal period $=27\frac{1}{3}$ days).

3. Find the angular diameter of the shadow cast by the earth at the moon's distance.

4. If R and r be the radii of the sun and the moon and D and d be their distances from the earth, show that the breadth of the moon's shadow on the earth where it falls perpendicularly $=\frac{2(rD-Rd)}{D-d}$

5. The shadow of the moon falls perpendicularly at a point on the earth's equator. Calculate the speed of the shadow in miles per hour, given that radius of the earth $=4000$ miles, radius of the moon's orbit $=230000$ miles, distance of the sun from the earth's centre $=93000000$ miles, lunation $=30$ days.

13. TIME

13.1. One kind of time, namely sidereal time, has been defined in SEC. **5.1.** It is regulated by the uniform diurnal motion of the First point of Aries. Sidereal time however is unsuitable for use in everyday life; for it may be the same hour by the sidereal clock at widely different times of the day. For instance, it will be 0 hr. 0 min. 0 sec. at noon on the 21st. March, in the morning on the 21st. June, at midnight on the 23rd. September, and in the afternoon on the 21st. December. Time to be convenient for use in everyday life must be more or less regulated by the sun.

Definition: The hour angle of the sun reduced to time on the basis that 24 hours is equivalent to 360° is called the *apparent solar time.*

Apparent solar time however would be somewhat unscientific. For the diurnal motion of the sun is not strictly uniform and the solar day (*i.e.*, the interval between two successive transits of the sun) is not constant on account of two reasons:

(1) Eccentricity of the sun's orbit, due to which the sun moves quicker at perigee and slower at apogee and generally at a non-uniform rate;

(2) Obliquity of the ecliptic to the equator, on account of which even if the sun moved uniformly *on the ecliptic*, his successive transits would not be at equal intervals.

The effect of Eccentricity and Obliquity of the sun's orbit will be analysed in the following section; but it will be realised even without the analysis that the sun himself is not entirely suitable for the regulation of time. So the following plan is adopted.

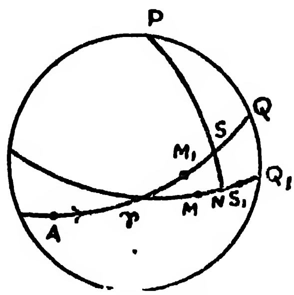

Fig. 13.1

Let AγQ be the ecliptic and γMNS$_1$Q$_1$ the equator, and A the perigee (Fig. 13.1). Let M_1 be an imaginary point which starts from A with the sun, moves uniformly and has the same period as the sun. When M_1 arrives at γ let a second imaginary point M start on the equator from γ with the same uniform motion. Diurnal motion of M will be strictly uniform; moreover it is never very much away from the sun. It is therefore entirely suitable for the regulation of time for everyday life.

Time regulated by the fictitious point, M, defined above, is called the *Mean solar time*; and the point itself is called the *mean sun*. To distinguish M_1 from M sometimes M_1 is called the *Dynamical Mean Sun* and M the *Astronomical Mean Sun.*

13.2. The difference between apparent solar time and mean solar time is always small and as defined below is known as the equation of time. Equation of time is thus an *interval* of time and is not an *equation* in the ordinary sense.

Equation of time is usually defined as follows:

Mean solar time = Apparent solar time + Equation of time ... ... (13.2a)

The equation of time is therefore positive or negative according to whether something is to be added to or subtracted from the apparent solar time in order to give mean solar time. Obviously the equation (13.2a) is equivalent to the following:

Hour angle of the mean sun = Hour angle of the true sun + Equation of time (Hour angles being measured in time) ... (13.2b)

Equation (13.2b) can be put in a little different form thus:

Hour angle of the Mean Sun = Sidereal time − R. A. of the Mean Sun.

Hour angle of the True Sun = Sidereal time − R. A. of the True Sun.

Hence the equation (13.2b) reduces to

R. A. of the true Sun = R. A. of the Mean Sun + Equation of time ... (13.2c)

Recently the nautical almanacs have adopted the following definition:

Hour angle of the sun = Hour angle of the mean sun + Equation of time (Hour angles being measured in time) ... (13.2d)

The equation of time so defined is the negative of that defined by the equations (13.2a) or (13.2b). In the following pages we shall adopt the older definition given by the equations (13.2a) or (13.2b).

In Fig. 13.1, let S be the true sun and S_1 a point on the equator such that

$\gamma S = \gamma S_1 = \odot$ = the longitude of the sun in time.

= the R.A. of the point S_1 in time.

$\gamma M = \gamma M_1 = l$ = the longitude of M_1 in time.

= the R.A. of the mean sun M in time.

$\gamma N = \alpha$ = the R.A. of the true sun S in time.

From equation 13.2c we have

E = R.A. of the True Sun—R.A. of the Mean Sun.

$= \alpha - l$ (Fig. 13.1) where E is the equation of time.

$= (\alpha - \odot) + (\odot - l)$ ... ... (13.2e)

Thus the equation of time consists of two components:

(1) The component $\alpha - \odot$ and (2) $\odot - l$. The component $\alpha - \odot$ is due to obliquity of the ecliptic—α differs from $\odot$ because the ecliptic is inclined to the equator; in our figure it $(= \gamma X - \gamma S_1)$ is negative. The component $\odot - l$ is due to the eccentricity of the path of the Sun—$\odot$ differs from l because on account of the eccentricity of the sun's orbit the sun does not have uniform angular motion; in our figure it $(= \gamma S_1 - \gamma M)$ is positive. The total, in the figure, is positive.

The value of the components can be calculated to any degree of approximation. A first approximation, sufficient for all practical purposes, is that the first component $= -10 \sin 2l$ minutes and the second $= 7 \sin (l + A\gamma)$ minutes.

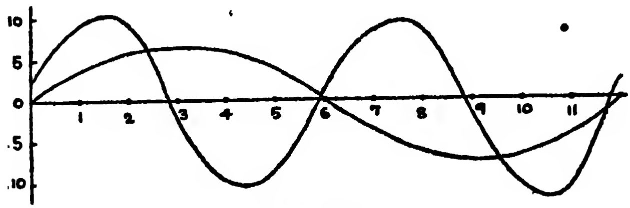

Fig. 13.2

In Fig. 13.2, the graphs of the two components are drawn against time which is proportional to l. It is to be noted that l is 0 on the 21st. March and $l + A\gamma$ is 0 on the 1st. January. The equation of time on any day can be obtained by adding the corresponding ordinates. The greatest positive value is 14 min. 28 sec. on the 11th February and the greatest negative value 16 min. 21 sec. on the 3rd November. *The value is 0 four times a year,* on about April 16, June 15, September 1, and December 25. These statements will no doubt be borne out by any accurate drawing of the graph. The second however can be seen to be true as follows. The component $-10 \sin 2l$ becomes -10 min., 10 min., -10 min., 10 min., on dates between (1) the 21st March and the 21st June, (2) the 21st June and the 23rd. September, (3) the 23rd. September and the 21st. December and (4) the 21st. December and the 21st. March respectively. Whatever the sign of the other component, since it is always less than 10 min. in magnitude, the sum of the two

components is negative, positive, negative, positive in succession on four dates in the year. Since the equation of time changes gradually, it must vanish while changing from positive to negative values or *vice versa*. Hence it vanishes four times a year.

13.3. *The sun-dial* is a device to indicate Apparent solar time. It consists of a straight rod ON, called Gnomon or Style, set in the direction of the north celestial pole through the centre O of a *horizontal* circle. The time is shown by the shadow of the rod cast by the sun on the circle. The circle may be graduated as follows:

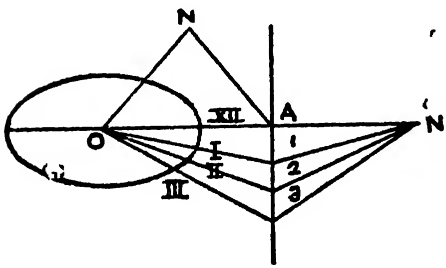

Fig. 13.3

Let OAN' be the line of section of the horizontal plane of the circle and the vertical plane through ON (Fig.13.3). Let NA be perpendicular to ON and N'A equal to NA. Let A123. . . . .be a straight line in the plane of the circle, perpendicular to the line OA. Draw N'1,N'2, ... , making the successive angles AN'1, 1N'2,. . . .each equal to 15°. Join OA, O1, O2. . . . . Put the marks XII, I, II, etc., successively at the intersections of the straight lines with the circle.

Imagine the plane of the paper to be folded about A12. . . . till N' coincides with N. The plane so folded is perpendicular to ON, and N'A, N'1, N'2,. . . .coincide with NA, N1, N2, ... The latter therefore also make successive angles of 15° with their neighbours. Now the plane of the hour circle of the sun rotates about ON at the rate 15° per hour. Hence NA, N1, N2, ...lie in the plane of the hour circle successively at intervals of one hour. It follows that the shadow of the rod ON will coincide successively with OA, O1, O2,. . . . .at noon, 1 P.M., 2 P.M.,. . . . . *i.e.*, at times agreeing with those indicated by the marks.

13.4. On account of the disparity between Apparent solar time and Mean solar time, morning and afternoon as reckoned by the clock may not be of equal duration. Morning is the period from sun-rise to 12 O'clock noon (mean time); and afternoon is the period from 12 O'clock noon (mean time) to sun-set.

To preserve continuity of the measure of time add 12 hours to the times shown by the clock in the afternoon; and let

T_m = time at sun-rise

T_n = time when the sun is on the meridian at noon

T_s = time at sun-set

E = the equation of time.

Then the length of the morning

$$=12-T_m=(T_n-T_m)-(T_n-12) \quad \ldots \quad (13.4a)$$

the length of the afternoon

$$=T_s-12=(T_s-T_n)+(T_n-12) \quad \ldots \quad (13.4b)$$

Since the equation of time E = (Mean time) − (Apparent time) at any instant, considering the times at apparent noon,

$$E=T_n-12$$

Moreover, $T_n-T_m=T_s-T_n$, since the period from sun-rise to apparent noon is equal to the period from apparent noon to sun-set. Hence subtracting equation (13.4a) from the equation (13.4b), we have

$$\text{(Afternoon)}-\text{(Morning)}=2(T_n-12)=2E \quad \ldots \quad (13.4c)$$

E may be taken to be the equation of time for the day because it does not change appreciably during a day.

13.5. The interval between two successive transits of the First point of Aries is called a *sidereal day*; and the interval between two successive transits of the Mean Sun is called a *mean solar day*. Now the mean sun completes one revolution of the equator, west to east, in course of a year relative to the First point of Aries *i.e.*, the latter completes one revolution east to west relative to the mean sun in the same time. But the mean sun completes 365¼ revolutions east to west with respect to the meridian in a year. Hence the First point of Aries completes 366¼ revolutions with respect to the meridian in a year. Hence

$$365\tfrac{1}{4} \text{ mean solar days} = 366\tfrac{1}{4} \text{ sidereal days} \quad \ldots \quad (13.5a)$$

This relation forms the basis of conversion of an interval of mean time to one of sidereal time and *vice versa*. The following table of conversion calculated from the equation (13.5a), facilitates computation.

1 hour of mean time = 1 hour 9.86 sec. of sidereal time.
1 min. of mean time = 1 min. 0.16 sec. of sidereal time.
1 sec. of mean time = 1 sec. of sidereal time. ... (13.5b)
1 hr. of sidereal time = 1 hr. −9.83 sec. of mean time.
1 min. of sidereal time = 1 min. −0.16 sec. of mean time.
1 sec. of sidereal time = 1 sec. of mean time.

Generally, m units of mean time

$$= (366\tfrac{1}{4}/365\tfrac{1}{4})m \text{ units of sidereal time}$$
$$= m + (1/365\tfrac{1}{4})m \text{ units of sidereal time}$$
$$= m + km \text{ units of sidereal time where } k \text{ stands for } 1/365\tfrac{1}{4}$$

Similarly s units of sidereal time

$=(365\frac{1}{4}/366\frac{1}{4})s$ units of mean time
$=s-(1/366\frac{1}{4})s$ units of mean time
$=s-k's$ units of mean time where k' stands for $1/366\frac{1}{4}$

(1) *To find sidereal time, given mean time, at Greenwhich.*

Before 1925, mean time was reckoned from mean noon in British Nautical almanac. But now it is reckoned from mean midnight; and the sidereal time at mean midnight is given in the almanac for every day.

Let m = the given mean time (reckoned from mean midnight)
s_0 = the sidereal time at the previous mean midnight (given in the nautical almanac)
s = the sidereal time required

The interval from mean midnight to the instant in question
$=m$ units of mean time
$=m+km$ units of sidereal time

Hence the sidereal time required $=s_0+m+km$ ...(13.5c)

(2) *To find mean time, given sidereal time, at Greenwich.*

Let s = the given sidereal time
s_0 = the sidereal time at the previous mean midnight (given in the nautical almanac)
m = the required mean time (reckoned from mean midnight)

The interval from the mean midnight to the instant in question $=s-s_0$ units of sidereal time
$=(s-s_0)-k'(s-s_0)$ units of mean time

Hence $m=(s-s_0)-k'(s-s_0)$ ... ... (13.5d)

(3) *To find the local sidereal time, given the local mean time,* at a place of longitude L° west of Greenwich.

Let m_1 = the given local mean time (reckoned from mean midnight)
m = the Greenwich mean time (reckoned from mean midnight) at the instant
s = the Greenwich sidereal time at the instant.
s_1 = the local sidereal time at the instant.
s_0 = the Greenwich sidereal time at the previous mean midnight

Let all the times be in hours.

$m=m_1+(1/15)L$, because mean midnight begins at Greenwich $(1/15)L$ hours earlier

$s=s_0+m+km$ ... by the equation (13.5c)
$=s_0+m_1+(1/15)L+km_1+k(1/15)L$

But $s=s_1+(1/15)L$, because the transit of γ takes place $L/15$ sidereal hours earlier at Greenwich

Hence $s_1=s_0+m_1+km_1+kL/15$ ... ... (13.5e)

(4) *To find the local mean time, given the local sidereal time, at a place of longitude L° west of Greenwich.*

With the notation of the previous paragraph,

$$m = (s - s_0) - k'(s - s_0) \text{ by the equation (13.5d).}$$

And $m = m_1 + L/15$, $\qquad s = s_1 + L/15$.

Substituting these values in the first equation,

$$m_1 + L/15 = s_1 + L/15 - s_0 - k'(s_1 + L/15 - s_0).$$

Or $m_1 = s_1 - s_0 - k'(s_1 - s_0) - K'L/15 \qquad \ldots \quad (13.5f)$

Example: To find the (local) sidereal time at New York at 10 hr. 25 min. 31 sec., local mean time, on the morning of September 1, 1931.

Given: longitude of New York is 74° W.

Sidereal time at mean midnight at Greenwich on September 1, (from nautical almanac)	22 hr.	36 min.	47 sec.
Mean time given at New York	10 hr.	25 min.	31 sec.
Add 74/15 hr. for west longitude (to convert to Greenwich mean time)	4 hr.	56 min.	
	15 hr.	21 min.	31 sec.
Add 15 × 9.86 sec., for 15 hr. (to convert to sidereal time)		2 min.	27.9 sec.
Add 21 × .16 sec., for 21 min. (to convert to sidereal time)			3.3 sec.
Sidereal interval elapsed since midnight at Greenwich	=15 hr.	24 min.	2 sec.
Sidereal time at mean midnight at Greenwich (given)	22 hr.	36 min.	47 sec.
	(24+14) hr.	0 min.	49 sec.
Subtract	24 hr.		
	14 hr.	0 min.	49 sec.
Subtract 74/15 hr., to convert to New York time	4 hr.	56 min.	
∴ Sidereal time at New York	=9 hr.	4 min.	49 sec.

13.6. *Definition:* The *Tropical year* is the period in which the sun completes a revolution in the ecliptic with respect to the first point of Aries.

Since the changes of seasons are due to changes of position of the sun relative to the first point of Aries, seasons repeat at intervals of a tropical year. The tropical year is equal to 365.2422 mean solar days approximately. The ancients employed the gnomon to determine the length of the tropical year; the interval between two transits of the sun when the shadow of the gnomon is exactly of the same size was noted; and it was divided by the number of years elapsed between the two observations.

Definition : The *sidereal year* is the period in which the sun completes a revolution in the ecliptic with respect to a point fixed among the stars.

On account of precession of equinoxes (SEC. 14.4) the First point of Aries moves backwards on the ecliptic with respect to stars at the rate of 50″.2 per year. So the sidereal year is a little longer, being 365 2564 mean solar days approximately.

Definition · The *anomalistic year* is the period in which the sun completes a revolution in the ecliptic with respect to the perigee.

The perigee advances along the ecliptic, *i e.*, moves eastwards with respect to stars, at a small rate 11″.25 per year; so the anomalistic year is the longest, being 365.2696 mean solar days approximately.

The tropical year is made the basis of the *civil year*, in order that seasons may repeat on regular dates: Necessarily the civil year must contain an integral number of days. According to the Julian calender, three years of 365 days are followed by one of 366, the latter being called a *leap year*. Accordingly, the average length of the civil year becomes 365.25 days. As this is too large by about .0078 days the accumulation of error in 400 years is a little over three days. A further correction, called Gregorian correction, cancels three leap years in four centuries. The two corrections, Julian and Gregorian, are incorporated in the following rule :- If the year be divisible by four it is a leap year; but if the last two digits be zero in the number indicating the year, it will be a leap year only if it be divisible by 400.

Gregorian calender was adopted in England in the year 1752. To make up for the accumulated error the day following September 2, 1752 was called September 14; the year also was taken to begin on January 1, instead of March 25.

Examples Worked Out

1. If at Greenwich h and h′ are the hour angles in degrees of the sun at t and t′ hours mean time, show that the equation of time at the preceeding mean noon expressed in fraction of an hour of mean time is $\dfrac{h't-ht'}{15(t'-t)}$

The rate of change of hour angle of the sun $=\dfrac{h'-h}{t'-t}$

The change in t hours $=\dfrac{t(h'-h)}{t'-t}$

Hence at previous mean noon, the hour angle of the sun is $h-\dfrac{t(h'-h)}{t'-t}$ degrees.

In other words, the equation of time is

$$\frac{t(h'-h)}{t'-t} - h = \frac{h't-ht'}{t'-t} = \frac{1}{15}\frac{h't-ht'}{t'-t} \text{ hrs.}$$

2. The shadow of a vertical rod at a place 4° west of Greenwich is observed to have the same length at 10h. 25m. A.M. and 14h. 5m. P.M. Greenwich time on a particular day. Find the equation of time on the day of observation.

The altitudes of the true sun at the instants when the shadows are equal are obviously equal. If T represents the Greenwich mean time of local apparent noon

$$T - 10h.\ 25m. = 14h.\ 5m. - T$$

$$\therefore\ T = \tfrac{1}{2}\ (14h.\ 5m. + 10h.\ 25m.)$$

$$= 12h.\ 15m.$$

Again Greenwich meantime of local mean noon
= 12h. 10m. (Since the place is 4° to the west of Greenwich).

Thus local mean noon is 1m. after local apparent noon.

Hence when local mean time is 12h. local apparent time is 12h. 1m.

∴ Equation of time on the day of observation = −1 m.

Exercise 13

1. Find the time at Vienna (longitude 16°20′ E) and at Chicago (longitude 87°40′ W) when it is 11 A.M. at Greenwich.
2. Where is a ship whose local mean time is noon and whose Greenwich chronometer reads 2 hr. 26 min. P.M.?
3. At mean noon on a given date the sidereal time is 3 hr. 30 min. What will be the sidereal time after 50 days at mean noon?
4. If on a particular day, morning be longer than afternoon by 5 min., what is the equation of time on the day?
5. The tropical year being 365.2422 mean solar days, how long will it be before a correction amounting to a day will be necessary over the Julian and the Gregorian correction to the year?
6. Find the sidereal time at 13 hr. 20 min. mean time, the mean time of transit of the first point of Aries being 21 hr. 40 min.
7. Find the time of the year if the mean time of transit of the first point of Aries be 21 hr. 40 min.
8. Find the mean time at 12 hr. 40 min. sidereal time, the sidereal time at the previous mean midnight of the place being 5 hr. 20 min.
9. A, B, C are certain places on the earth. At one and the same instant the clocks of the several places indicate respectively 1 A.M., 2 A.M., 3 A.M. ..... Prove that the longitudes of A, B, C .... are in A. P. (C. U. 1920).

10. If T, S and A be the lengths of the tropical, sidereal and anomalistic year respectively, show that

$$\frac{A-T}{S-T}=1.224 \text{ approximately}$$

Given that the perigee advances by 11."25 per year; and the precession of equinoxes is 50."26.

11. If y be the length of the tropical year, s the length of the sidereal day and m the length of the mean day, prove that

$$\frac{1}{s}-\frac{1}{m}=\frac{1}{y}$$

14. ABERRATION, PARALLAX, PRECESSION AND NUTATION

14.1. Bradley discovered the phenomenon of aberration while making observation on γ Draconis in search of stellar parallax. He chose the particular star because it was fairly bright and presumably near; at the same time it passed close to his zenith, so that meridian observations were little affected by refraction. He discovered a change of position within a short time; but it was not according to the theory of parallax. This led him to an entirely different explanation and the phenomenon was named Aberration.

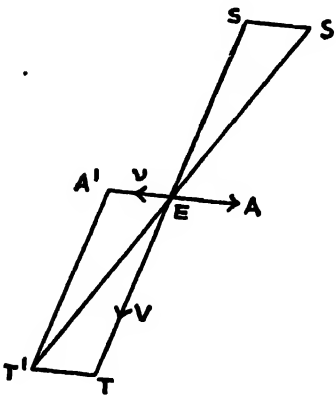

Fig. 14.1a

Let E be the earth and S a star (Fig. 14.1a). If the earth were at rest, light from the star travelling with velocity V, say, would reach E in the direction SE; consequently the star would be seen in the direction ES. If however the earth be moving with the velocity v in the direction EA, we should perceive only the relative velocity of light, not the actual. Produce SE to T making ET=SE. Let ET represent V. Produce AE to A′ and let EA′ represent v, the velocity equal and opposite to that of the earth. The relative velocity of light is the resultant of the two velocities represented by ET and EA′ and is therefore represented by the diagonal ET′ of the parallelogram ETT′A′. Draw SS′ parallel to EA to meet T′E produced at S′. The star will therefore be seen in the direction ES′, displaced from its true direction ES. The angle SES′ by which the star is displaced is briefly called its *aberration*.

To find the amount and direction of aberration.

Employing Fig. 14.1a, the aberration of S is $\angle SES' = \alpha$, say. From the triangle ETT′,

$$\frac{\sin TET'}{\sin ETT'} = \frac{TT'}{ET} = \frac{v}{V} \quad \dots \quad \dots \quad (14.1a)$$

$\angle TET' = \angle SES' = \alpha$; and $\angle ET'T = \angle A'ET' = \angle S'EA = E$, say.

The angle E is called the *earth's way*. The equation (14.1a) becomes

$$\frac{\sin \alpha}{\sin E} = \frac{v}{V},$$

$$\text{Or since } \alpha \text{ is small, } \alpha = \frac{v}{V} \sin E \quad \dots \quad \dots \quad (14.1b)$$

where α is in radians. Expressed in seconds of arc, the relation is $\alpha'' = k'' \sin E = 20''.5 \sin E$ nearly ... (14.1c)

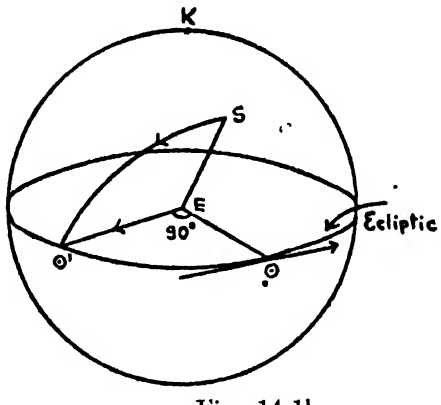

Fig. 14.1b

It will be noted from Fig. 14.1a that the displacement due to aberration is towards the point on the celestial sphere towards which the earth is moving at the instant. In Fig. 14.1b let ⊙⊙′ be the ecliptic, S a star, K the pole of the ecliptic and E the earth, at the centre of the sphere. Let ⊙ be the position of the sun at any time; the tangent to the ecliptic at ⊙ indicates the apparent direction of motion of the sun at the time. Hence E⊙′ drawn parallel and in the opposite sense to the forward direction of the tangent indicates the motion of the earth. *The displacement due to aberration is therefore towards ⊙′, which is 90° behind the sun on the ecliptic.*

To examine the course of a star's displacement during a year, on account of aberration.

Assume that the orbit of the earth round the sun is a circle. Then in Fig. 14.1a, A the end of the straight line EA, representing the velocity of the earth, describes a circle round E in course of a year. The displaced position of the star, S′, likewise describes a circle round the mean position S, parallel to the

plane of the ecliptic, as represented by the small oval round S in Fig. 14.1c.

Let the cone described with E as vertex and the circle S as base be produced to meet the celestial sphere; the section—because it is of a cone with a circular base—must be an ellipse. Since we see celestial objects projected on the celestial sphere, this ellipse is the apparent course of displacement due to aberration. Let S, the mean position of the star, be projected to σ on the celestial sphere. The semi-major axis of the ellipse = the maximum displacement due to aberration = k of equation (14.1c), along $\sigma\odot'$, where $\sigma\odot' = 90°$. Let K be the pole of the ecliptic. Then $K\odot = 90°$. It follows that $\odot'$ is the pole of $K\sigma$ and so $\sigma\odot'$ is perpendicular to $K\sigma$. That is to say, the major axis is parallel to the ecliptic. The semi-minor axis = the minimum displacement = k sin $(90° - K\sigma)$ = k sin β (where β = latitude of the star) along the extension of $K\sigma$; for $\sigma\odot'$ is obviously least when it is perpendicular to the ecliptic. It follows that

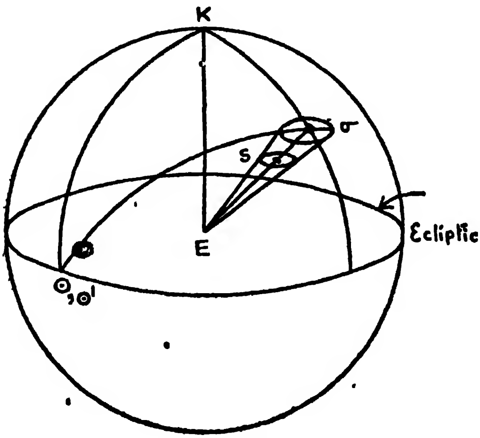

Fig. 14.1c

$$\text{major axis} : \text{minor axis} = 1 : \sin\beta$$

Summarising the effect of aberration on the position of a star, we may therefore state as follows:

(1) All stars aberrate towards a point on the ecliptic 90° *behind* the sun.

(2) The amount of aberration is k'' sin E where E is the earth's way, and k'' is the same for all stars and equal to $20''.5$ approximately.

(3) In course of a year a star of latitude β seems to describe, due to aberration an ellipse on the celestical sphere such that the major axis : the minor axis = 1 : sin β the major axis being parallel to the plane of the ecliptic.

On account of the rotation of the earth about its axis, the observer on the surface of the earth has a velocity which gives rise to what is called *Diurnal aberration.* The velocity (about 1/3 mile per sec. for a point on the equator) being much smaller

than the velocity of the earth in its orbit (about $18\frac{1}{2}$ miles per sec.), the amount of diurnal aberration is very small indeed.

14.2. *Definition:* The *annual parallax* of a celestial body (which should be distinguished from diurnal parallax defined in SEC. 11.1) is the angle subtended at it by the line joining the sun and the earth.

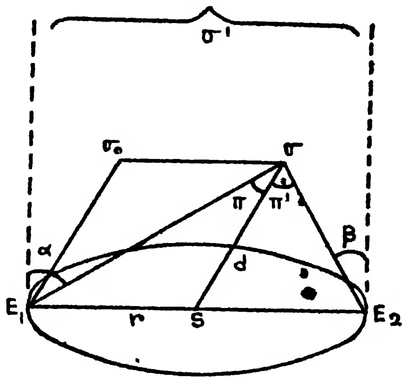

Fig. 14.2a

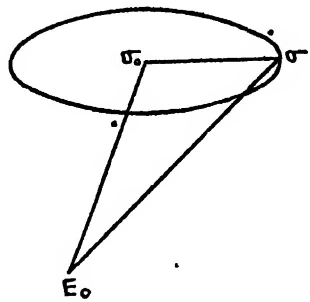

Fig. 14.2b

In Fig. 14.2a, let S be the sun, E_1 a position of the earth in its orbit and σ a star. The angle $E_1\sigma S=\pi$, say, is the parallax of the star corresponding to the position E_1 of the earth. Let $r=SE_1$ be the radius of the earth's orbit and $d=S\sigma$ be the distance of the star from the sun. Then from the triangle $SE_1\sigma$

$$\frac{\sin \pi}{\sin \sigma E_1 S}=\frac{r}{d}$$

Or $\sin \pi=(r/d)\sin \sigma E_1 S$; since π is small, we have the approximate formula $\pi=(r/d)\sin\sigma E_1 S$ ... (14.2a)

where π is in radians. The maximum value of π, say π_0, is given by $\pi_0=r/d$; or $\pi_0''=(r/d)\ 206265''$ ... (14.2b) if expressed in seconds of arc. Equation (14.2a) may now be written as $\pi=\pi_0 \sin E$ ... ... (14.2c)

where E is the elongation of the star.

In Fig. 14.2b, suppose the apparent directions of the star to be drawn from a fixed point E_0, representing the earth. Draw $E_0\sigma_0$ parallel and equal to $S\sigma$ of Fig. 14.2a and $\sigma\sigma_0$ parallel to SE_1 of Fig. 14.2a. It follows that $\sigma_0\sigma$ in Fig. 14.2b is equal to the radius of the earth's orbit. Hence σ in Fig. 14.2b, describes a circle round the mean position σ_0 as centre. The amount of displacement from the mean position $=\angle\sigma_0E_0\sigma=\angle E_1\sigma S=\pi=\pi_0\sin E$ (equation 14.2c) *i.e.*, the displacement varies as the sine of the elongation of the star.

Also it is towards the sun, while that due to aberration, as we have already shown, is towards the point 90° behind the sun.

By the parallax of a star, we usually mean its maximum value; and when it is known, the distance of the star can be found from equation (14.2b). Stellar distances being enormous, the units employed in their measurement is either (1) *parsec* or (2) *light-year.*

One *parsec* is the distance at which the annual parallax of a star would be one second of arc.

One *light-year* is the distance which light takes one year to travel at the rate of 186000 miles per second.

The relation between the two units, namely 1 parsec=3.26 light-years, can be easily obtained and is left to the reader as an exercise.

It is to be noted that the distance in parsecs of a star is the reciprocal of its parallax in seconds of arc; for

$$\pi_0'' = 206265\frac{r}{d} \text{(Equation 14.2b)}$$

$$\therefore \quad 1 \text{ parsec} = 206265 \text{ r}$$

$$\text{Hence } \pi_0'' = \frac{1}{d} \text{ or } d = \frac{1}{\pi_0} \text{ parsecs.}$$

We have a further unit of distance: The semi-major axis of the earth's orbit is known as *Astronomical unit of distance.* It follows that

1 parsec=206265 astronomical units.

Bessel (German astronomer: 1684-1846.) in his search for parallax chose the star 61 Cygni on account of its large proper motion, which he presumed to be a sign of nearness, and succeeded in obtaining its parallax by the "differential method". In fact he was the first to find any stellar parallax and so first to dispose of a serious criticism of the Copernican hypothesis.

Differential method (or Bessel's method) of finding stellar Parallax:

Let E_1 and E_2 (Fig. 14. 2*a*) be two positions of the earth in its orbit, at an interval of six months. Observe the small angles between the star in question and a very faint star σ' in its neighbourhood from E_1 and E_2 respectively. The faint star is presumed, on account of its faintness, to be immensely distant in comparison. Its directions from E_1 and E_2 should therefore be parallel. Now,

$$\angle E_1\sigma S = \pi = \pi_0 \sin \sigma E_1 S, \text{ and } \angle E_2\sigma S = \pi' = \pi_0 \sin \sigma E_2 S$$

$$\text{Therefore } \pi + \pi' = \pi_0(\sin \sigma E_1 S + \sin \sigma E_2 S)$$

$$\text{Or } \pi_0 = (\pi + \pi')/(\sin \sigma E_1 S + \sin \sigma E_2 S)$$

But $\pi+\pi'=\angle\sigma' E_2 \sigma+\angle\sigma' E_1 \sigma=\alpha+\beta$, where α and β are the observed angles between the faint star and the star in question. So, $\pi_0=(\alpha+\beta)/(\sin\sigma E_1S+\sin\sigma E_2S)$ . ... (14.2d)

All the quantities on the right hand side are therefore derived from direct measurement, so that π_0 becomes known.

Instead of direct measurement, photographic method is now available for obtaining α and β.

Absolute method of finding stellar parallax:

The method consists of a comparison of positions of the star in question, observed at different times of the year, after all possible corrections have been made. The difficulty is that the corrections themselves are always larger than the parallax, and any slight error in the former may completely mask the latter. Even seasonal variations of the instruments become of importance in such delicate observation. For these reasons stellar parallax by absolute method is never reliable. In spite of the theoretical objection that it gives only a relative value, the differential method is in practice the most reliable.

Although geometrical methods form the basis of our knowledge of stellar distances, new approaches have made it possible to estimate distances far beyond their capacity. Derivation of the distance of a *binary system*, explained below, provides an instance in point.

A binary system is a pair of very close stars which are found to revolve about each other. Suppose the line of sight of such a pair of stars is in the plane of its orbit. Spectroscopic method (already described in SEC. 7.2) yields the velocities of the components along the line of sight. Hence on the hypothesis that the orbit of each about their centre of gravity is circular the measure of the radius can be obtained from the period of revolution. The distance between the components in *angular* measure can also be observed. So the parallax of the pair is obtained.

Different types of binary systems will be described and the question of parallaxes of binaries in general will be touched upon in SEC. 15.2.

14.3. *To examine the course of displacement of a star during a year on account of parallax.*

Let the little circle at S in Fig. 14.1c be the relative path of the star due to displacement of parallax. Obviously its plane is parallel to that of the ecliptic.

Let the cone, described with E as vertex and the circle at S as base, be produced to meet the celestial sphere; the section —because it is of a cone with a circular base—must be an ellipse. Since we see celestial objects projected on the celestial sphere, this ellipse is the apparent course of displacement due to

parallax. Let S, the mean position of the star, be projected to σ on the celestial sphere. The semi-major axis of the ellipse = the maximum displacement due to parallax = π_0 of equation (14.2c), along $\sigma\odot$ where $\sigma\odot$ (elongation of the star) = 90°. Let K be the pole of the ecliptic. Then K$\odot$ = 90° It follows that $\odot$ is the pole of Kσ and so $\sigma\odot$ is perpendicular to Kσ. That is to say, the major axis is parallel to the ecliptic. The semi-minor axis = the minimum displacement = $\pi_0 \sin (90° - K\sigma) = \pi_0 \sin \beta$ (where β = latitude of the star), along the extension of Kσ; for $\sigma\odot$ is obviously least when it is perpendicular to the ecliptic. It follows that

major axis : minor axis = 1 : sin β

The effect of parallax on the apparent position of a star is therefore as follows, and may be contrasted with that of aberration listed at the end of SECTION 14.1.

(1) A star is displaced towards the sun on account of parallax.

(2) The amount of parallax is π_0 sinE where E is the elongation of the star and π_0 is a constant *different for different stars.*

(3) A star due to parallax seems to describe an ellipse on the celestial sphere such that the major axis : the minor axis = 1 : sin β; and the major axis is parallel to the ecliptic.

As there is close analogy between the effect of aberration and annual parallax it is instructive to compare them in detail.

Points of similarity:

(1) The locus of the apparent position of a star due to both aberration and annual parallax is an ellipse.
(2) The eccentricities of both the ellipses are the same.
(3) The major and the minor axis of both the ellipses are respectively parallel and perpendicular to the ecliptic.

Points of difference:

(1) A star is displaced towards the sun by annual parallax but towards a point 90° behind the sun by aberration.
(2) Annual parallax of a star depends on its distance but the coefficient of aberration is the same for all stars.

14.4. *Precession:* The R.A. and the Decl. of a star undergo *periodic* changes on account of aberration and parallax. Observations at long intervals show that they also undergo *progressive* changes over and above the periodic. The mean positions of stars among themselves and the position of the ecliptic among them are however found not to alter. The obliquity of the ecliptic has really a periodic change, which will be considered in a little greater detail in the next section. We may at present take it to be constant and equal to its average value. The progressive changes in the R.A. and the Decl. of a star are therefore due to a shifting of the equator so that its

inclination to the ecliptic remains the same on the average. Moreover the shifting is such that the first point of Aries and the first point of Libra move *backwards* on the ecliptic. The equinoxes therefore recur before the sun has completed 360°, on the ecliptic *i.e.*, in a sense before their due time. The phenomenon is therefore known as the *Precession of equinoxes.* Geometrically, the backward motion of the first point of Aries is the same as the rotation of the celestial pole round the pole of the ecliptic in the proper direction.

Physical explanation of the phenomenon of Precession of equinoxes.

The phenomenon is principally due to the unequal attraction of the sun and the moon on the equatorial bulging portion of the earth. It is therefore called luni-solar precession. Below we consider the effect of the sun's attraction only, though the moon being nearer to the earth is responsible for a larger share of the precession.

In Fig. 14.4a, let the spheroid, centre O, represent the earth and S the sun. Let BOA represent the equatorial plane of the earth; OS of course is in the plane of the ecliptic.

The earth may be considered to be made up of two portions: (1) the inner spherical portion and (2) the protuberent equatorial belt. The attraction of the sun on the inner sphere is a force F, say, through the centre O. The attraction on the *nearer* bulging portion is a force F_1 along AS, say; that on the *farther* bulging portion is a similar force F_2 along BS, say. The force F_1 is obviously larger than the force F_2; but even if they were equal, their resultant would pass through a point on the equatorial plane nearer to A than B. It follows that the resultant of F_1, F_2 and F is a force F_0 through O′ on the equatorial plane which is on the same side of O as S. The resultant force F_0, along O′S therefore tends to turn the plane of the equator towards the plane of the ecliptic.

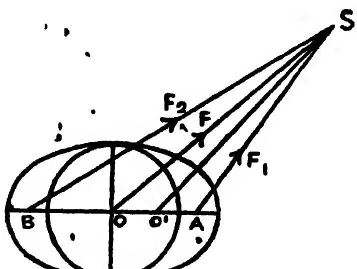

Fig 14.4a

Let K and P represent the poles of the ecliptic and the equator respectively (Fig. 14.4b). The earth rotates west to east about its axis *i.e.*, in the direction of the arrowhead on the equator in our figure. This angular motion should, according to the usual right-hand screw rule, be represented by OP which is perpendicular to the plane of

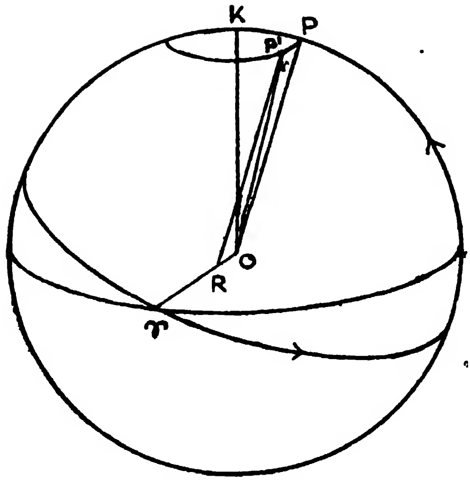

Fig. 14.4b

rotation and is also in the proper sense. The resultant force of attraction of the sun, F_0, generates an angular motion on the earth, which has the effect of tending P to move towards K. This should be represented, according to the same rule, by a length OR along $O\gamma$. By the parallelogram law, the two angular motions have a resultant in the direction OP′, where PP′, which is parallel to OR, is easily seen to be perpendicular to the arc KP. Since P moves always perpendicular to KP it describes a small circle round K; so the phenomenon of precession of equinoxes is explained.

The attraction of the planets causes a small change in the position of the ecliptic, and so contributes a small factor towards precession. The total, due to all the causes combined, is called the '*General precession* of which the average amount is 50″.2 per year.

14.5. *Nutation:* After allowing for aberration, Bradley found that there was still a residual variation in the declination of the star γ Draconis, which he had been observing; and the period of the variation was 18⅗ years, exactly the period of revolution of the moon's nodes. The periodic variation of the declination could only be attributed to a periodic movement of the celestial pole *i.e.*, to a sort of *nodding* of the celestial pole. This phenomenon was therefore named *Nutation* which literally means nodding. The coincidence of the two periods led him to the true explanation of the phenomenon.

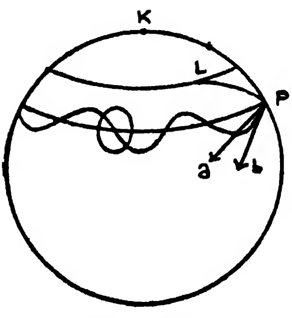

Fig. 14.5

Explanation of the Phenomenon of Nutation:

Let L be the pole of the moon's orbit (Fig. 14.5). Since the inclination of the moon's orbit to the ecliptic remains practically constant but her nodes revolve once in 18⅗ year, L revol-

ves round K, the pole of the ecliptic, once in $18\frac{2}{3}$ years. While the attraction of the sun causes the celestial pole P to move in the direction Pa perpendicular to KP, that of the moon similarly causes it to move in the direction Pb perpendicular to LP. The resultant motion on calculation is found to be a combination of (1) a periodic motion of P in an ellipse of which the major axis is 9″ along KP and the minor axis is 6″, perpendicular to KP and the period is $18\frac{2}{3}$ years; and (2) a progressive angular motion, round K, of the centre of the ellipse at the rate of 50″.2 per year.

The first component of the motion, namely the periodic motion round the mean position accounts for nutation. The two motions combined give to P a resultant motion along a wavy curve as shown in the figure.

14.6. In this chapter we have seen that the observed R.A. and Decl. of a star, supposed already corrected for refraction, are further affected by (1) Precession and Nutation, (2) Aberration and (3) Annual parallax. In the majority of cases, the annual parallax is inappreciable; and, in order to compare positions of a star at different times, reduction on account of the first two causes only need be carried out. If the comparison is to be made within the interval of a year, we refer the positions to the equator and the first point of Aries at the beginning of the year. For comparison at longer intervals, the changes of the positions of the equator and the first point of Aries should be taken into account. Such comparisons of positions of stars at long intervals have revealed in many cases the existence of progressive changes over and above those we have so far considered. Such changes are known as *proper motion.*

Let the mean position of a star (*i.e.*, the position which would be observed from the sun) be referred to the equator and the first point of Aries at the beginning of the year. The quantities to be added to the mean R.A. and Decl. to obtain the observed R.A. and Decl. at any time during the year can be, we assume, expressed mathematically as follows:

$$\text{Observed R.A.} = \text{Mean R.A.} + Aa + Bb + Cc + Dd + \mu_a$$
$$\text{Observed Decl.} = \text{Mean Decl.} + Aa' + Bb' + Cc' + Dd' + \mu_b \qquad \dots \quad (14.6)$$

The terms Aa, Bb and Aa′, Bb′ proceed from the effect of precession and nutation; and Cc, Dd and Cc′, Dd′ from that of aberration; and μ_a and μ_b are terms due to the proper motion defined above. The quantities A, B, C, D are independent of the co-ordinates of the star and depend only on the time in question; they are called *Bessel's Day numbers.* The logarithms of these quantities for each day are tabulated in the nautical almanac. a, a′, b, b′, c, c′, d, d′, on the other hand, depend only on the co-ordinates of the star and not on time; they

have to be calculated for individual stars, for the purpose of reduction of the observed position.

Examples Worked Out

1. Where must a star be situated so that its annual parallax may be (i) maximum, (ii) minimum? (C.U. 32).

The parallax p is given by $p=(r/d) \sin E$, where E is the angle between the direction of the star and that of the sun. [Eq. (14.2a)]

$\therefore$ it is maximum when E is 90° and minimum when E=0° or 180°.

2. Find where a star should be so that the effect of aberration on it is annulled by the effect of refraction.

Let Z be the zenith, and A the position of the point on the ecliptic, 90° behind the sun. At the instant under consideration the position of the star will be S on the arc joining Z and A such that $K \tan ZS = k \sin AS$, where K and k are the coefficient of refraction and the constant of aberration respectively.

3. If at midnight on the shortest day z be the zenith distance of a star such that in this position the aberration is entirely counteracted by refraction, show that z is given by an equation of the form $\sin^2 z + k \sin z = 1$ (C.U., Hon., 1930).

On the shortest day (*i.e.*, 21st December) R.A. of the sun is 270°. At midnight on that day the first point of Aries will therefore be at the west point and the point A towards which stars aberrate will be coincident with the east point. If at the instant under consideration S be the position of a star on the arc ZA such that the effect of aberration on it is entirely counteracted by that of refraction

$K \tan ZS = k' \sin AS$ where K and k′ are the coefficient of refraction and constant of aberration respectively.

But $ZS = z$ and $AS = 90°\text{-}z$

$$\therefore\ K \tan z = k' \sin (90° - z) = k' \cos z$$

$$\text{Or}\quad K \sin z = k' \cos^2 z = k'(1 - \sin^2 z)$$

$$i.e.\quad \sin^2 z + \frac{K}{k'} \sin z = 1$$

$$i.e.\quad \sin^2 z + k \sin z = 1 \text{ where } k = \frac{K}{k'}$$

4. Show that the locus of all stars which have no aberration in right ascension at any given time is a great circle.

Let S be the true position of a star and S′ its apparent position as affected by aberration. Since there is no aberration

in right ascension the great circle passing through S and S′ will be perpendicular to the equator and will therefore pass through the pole of the equator. It will also pass through the point on the ecliptic 90° behind the sun. Hence at any time all stars lying on the great circle through the pole of the equator and the point 90° behind the sun on the ecliptic will have no aberration in right ascension.

5. Prove that a star has no precession in declination if its R.A. is either 90° or 270° (Andhra Univ. '44).

Let P be the celestial pole and K the pole of the ecliptic. Due to precession P moves round K in a small circle. Hence the precession in declination is 0 when P moves perpendicular to the declination circle. Therefore KP must be coincident with the declination circle; so the R.A. is either 90° or 270°, for the pole of KP is the first point of Aries.

Exercise 14

1. Find the positions of all stars which are displaced by the same amount by aberration at any given time. How many of these stars will be displaced by the same amount at another given time?
2. Show that the parallax of a star on the ecliptic is maximum when the aberration is minimum; and vice versa. When is the aberration of the pole star maximum? when minimum? (C U. 1927).
3. The co-ordinates of a star are: R.A.$=$6 hr., Decl.$=$0. When is its (i) aberration (ii) parallax maximum and minimum?
4. Calculate the co-efficient of aberration due to the rotation of the earth on its axis, for an observer on the equator of the earth. Compare it with the co-efficient of aberration due to the earth's motion in its orbit. (Take velocity of the earth in its orbit$=$20 miles per sec., radius of the earth$=$4000 miles, velocity of light$=$186000 miles per sec.)
5. If p be the number of seconds in the annual parallax of a star, show that its distance is approximately 16/(5p) light-years. What is the distance in parsec? (Pat. 1948).
6. If p be the number of seconds in the parallax of a star, and 8.″8 is the parallax (diurnal) of the sun, find the distance of the star in miles. (The radius of the earth$=$4000 miles. Log. table may be used).
7. At the solstices show that a star on the equator has no aberration in declination (Andhra 1941).
8. Show that, owing to aberration, a star at the pole of the ecliptic appears to describe a circle round its true position in course of a year. (C.U. '35, '39, '47. Dac. '35).

   Show also that, due to aberration, a star on the ecliptic appears to oscillate to and fro in a straight line in course of a year. (Nagpur. 1940).

15. STELLAR ASTRONOMY

15.1. It has been remarked in SEC. 2.1 that the ancients regarded stars as fixed. Halley, in 1718, first detected changes of positions of a few stars by comparing observations of his own time with those of the ancients. Today there have accumulated abundant observational data of such changes. The sun—in reality a star—is also in motion, as we shall see presently. Let O be the mean position of the observer (*i.e.*, the sun) (Fig. 15.1). Let S and S′ be the relative positions of a star at the beginning and the end of an interval, say, a year. Draw S′M perpendicular to OS. SM and S′M are the displacements along and perpendicular to the line of sight. The latter causes an angular displacement which is called *Proper motion* and of which μ_a and μ_b of equation (14.6) are the components. The radial displacement SM can not be directly perceived. It can however be measured by the spectroscopic method explained in SEC. 7.2. Proper motion, as already mentioned in SEC. 14.6, is measured by direct observation of positions at intervals of, say, 50 years; or better by comparing stellar photographs at intervals of a shorter period, say, 20 years. Theories of motions of stars as a system have been built on the accumulated data of both proper motions and radial velocities. We shall take them up presently. First, we shall, in the next two sections, deal with binaries and variable stars, the study of which help us to extend our knowledge of stellar distances.

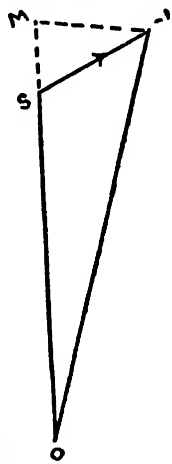

Fig. 15.1

15.2. Sometimes a star appearing single to the naked eye is resolved into a pair in the telescope. Such stars, and generally two stars found very close together, are called *Double stars*. Double stars may not be physically connected: their lines of sight may be close only by accident. It should however be remarked that the probability of such closeness due to chance can be calculated and is found to be very small.

In the cases of many pairs however, each star has been definitely found to revolve relatively in an orbit round the other. Obviously they are under mutual gravitational influence. The period of revolution is usually several years and considerable time is required for observation. Pairs for which the periods are still longer may have escaped notice on account of the short time they have been under observation and may be wrongly

supposed not to be physically connected. A pair of stars which have been definitely found to move round each other under mutual gravitation is called *Binary stars* or simply a *Binary*.

Let m_1 and m_2 be the masses of the two components of a binary, r_1 and r_2 their mean distances from the common centre of mass G and P the periodic time.

Fig. 15.2

We deduced equation (8.6a) on the assumption that the orbit of the earth round the sun is a circle. The same relation, it is known, holds for elliptic orbits if R is the *mean* distance apart. For the binary system therefore we have the similar equation

$$m_1 + m_2 = \frac{4\pi^2}{G'} \frac{(r_1 + r_2)^3}{P^2}$$

where G′ is the constant of gravitation ... (15.2).

The plane of the orbit of a binary is rarely perpendicular to the line of sight; the observed orbit is usually an oblique projection of the actual, on the celestial sphere. The true orbit can be mathematically obtained from a sufficient number of observations, though the scale is not known till the parallax is known. In a few cases parallaxes have also been obtained; so in equation (15.2) above both $r_1 + r_2$ and P are known. $m_1 + m_2$ therefore follows. It has been found that the sum is of the order of twice the mass of the sun. Conversely, when the parallax of a binary is unknown, on the assumption that $m_1 + m_2 = 2 \times$ mass of the sun, an estimate of $r_1 + r_2$, and so, the parallax of the system, follows from the equation (15.2).

Certain binaries are not resolved into their components even in a telescope, but their binary character and full information regarding their orbits are revealed by spectroscopic methods. Such binaries are called *spectroscopic binaries.*

15.3 There is an interesting class of stars whose apparent brightness does not remain constant. They are called *variables.* In one of the different types of variables the light-curve *i.e.*, the graph showing the variation of observed brightness, is extremely regular. An example is the light curve of Algol, represented in Fig. 15.3a. Variables of this type have been interpreted as binaries one of whose components is dark and periodically eclipses the other which is bright. The dimensions of the components as well as the particulars of the orbits can be specified so as to explain fully the variation of light. Such variables are called *Eclipsing variables.*

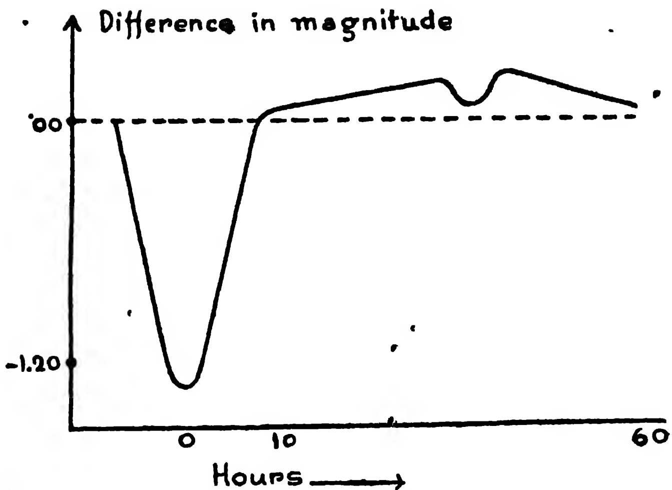

Fig. 15.3a*

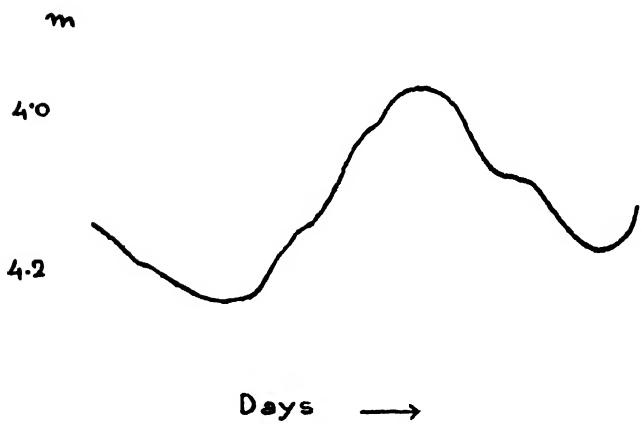

Fig. 15.3b*

There is a second type of variables known as *Cepheid variables*; the light-curve of δ Cephei, represented in Fig. 15.3b, is typical of this class. The variation of light of the Cepheid variables is believed to be due to pulsation *i.e.*, periodic expansion and contraction of the star itself. The consideration of Cepheid and other types of variables properly belongs to Physical Astronomy and is beyond the province of our discussion. We

* Figs. 15.3a and 15.3b do not represent accurate graphs; they are to convey a general notion of the changes taking place.

should only mention that there has been established a period-luminosity relation among the Cepheid variables (here luminosity means absolute brightness, in other words, brightness when at a standard distance) which enables us to find distances of systems containing Cepheids at incredible depths of the universe.

15.4 From the proper motions known in his time, Sir William Herschel observed that stars in one region of the sky are 'spreading out' and those in the diametrically opposite region "closing in". He concluded that the effect was due to the motion of the sun towards one region and away from the other. The point of the sky towards which the sun is moving is called the *solar apex*; and the opposite point the *solar antapex*. The relative displacement of a star due to solar motion (and not due to the star's own motion) is called *Parallactic displacement*.

Presumably, stars possess individual motions over and above the parallactic. Assuming individual motions to be random, their effect is eliminated by combining the proper motions of several stars in any small region, so that only the parallactic displacement stands out in the average. But for such averaging, stars should be chosen judiciously. For, groups of stars often possess common motions; the calculated parallactic displacements and therefore the derived position of the solar apex will be biased if a disproportionately large number of stars of a group enter into the solution. Instead of proper motions of stars their radial velocities may also be employed for the calculation of solar motion; and both kinds of data yield the same result; namely, the co-ordinates of the solar apex are R.A. $=270°$, Decl. $=30°$, and the velocity of the sun is 13 miles per second.

It should be remarked that the solar motion so obtained is with reference to the centre of mass of the stars employed in the calculation. Such stars are really members of the *local cluster i.e.*, of the group in the immediate neighbourhood of the sun. We shall learn further on that this cluster has an enormous velocity with reference to the centre of the Galaxy which constitutes the entire universe of stars, barring systems called extra-galactic. Extra-galactic systems are characterised by their enormous distances and surprisingly large velocities.

Solar motion, being the individual motion of one star among many, is not of particular interest by itself; its importance arises from the fact that it is needed in the reduction of observed motions of stars in general.

15.5. Considering stars in the sky as a whole, it is at once apparent that there is marked concentration towards the Galaxy. The cloud-like appearance of the Galaxy itself has been resolved in the telescope into a dense crowd of very minute stars which are presumably very distant. The Herschels, father and son, made extensive count of stars in all parts of the sky, and the following table is based on their observations.

TABLE SHOWING CONCENTRATION OF STARS TOWARDS THE GALAXY

North galactic latitude zones	Average number of stars per field of 15′ diameter
90° to 75°	4.32
75° to 60°	5.42
60° to 45°	8.21
45° to 30°	13.61
30° to 15°	24.09
15° to 0°	53.43
0° to −15°	59.06
−15° to −30°	26.29
−30° to −45°	13.49
−45° to −60°	9.08
−60° to −75°	6.62
−75° to −90°	6.05

It is estimated—assuming that greater concentration means that stars spread out to a greater distance—that the galactic system of stars is of the shape of a very oblate spheroid, the larger diameter of which may be eight or ten times the smaller. The northern galactic pole is estimated to be at R.A. = 190° and Decl. = 28°. The sun is situated a little north of the central plane and his distance from the centre may be of the order of 40000 light-years. The distances of *globular clusters* (*i.e.*, spherical formation of stars frequently met with in the sky) can be found by indirect methods, such as that mentioned in SEC. 15.3 and come out to be enormous. Obviously they do not belong to our local cluster and do not pertake of its motion. It is therefore possible to estimate the speed of the local cluster by taking the average of the observed speeds of the globular clusters; for, on averaging, the individual speeds of the globular clusters are eliminated. In this way it has been estimated that the speed of our local cluster is of the order of 150 miles per second.

15.6. We now take up the question whether this enormous galactic system of stars has any motion as a whole.* The answer is given by the works of J. H. Oort. Oort based his results on radial velocities of stars. Observation of radial velocity is possible even for very distant stars and can be carried out with high accuracy. The radial velocity is also immediately available and one is not required to wait for a number of years as in the determination of proper motions. Assume that there is rotation of the galactic system about a centre considerably distant from us. Since stars obviously do not revolve as a solid body, their

velocities will decrease with distances from the centre of rotation according to the law of gravitation [*cf.* equation (8.5)].

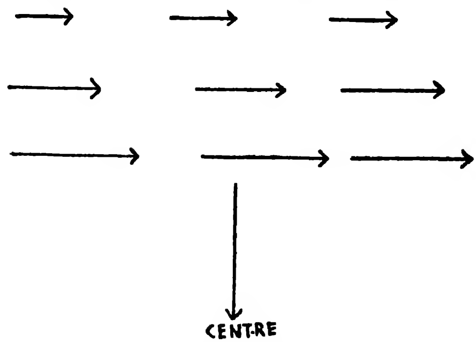

Fig. 15.6a

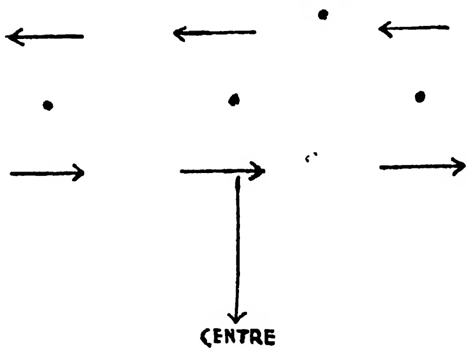

Fig. 15.6b

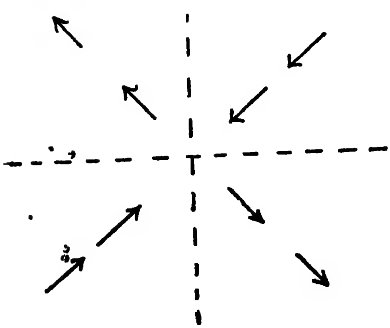

Fig. 15.6c

Let Fig. 15.6a represent the actual velocities of stars, by the lengths of the straight lines, at different distances from the centre. The centre is very far away in the direction of the arrowhead. Fig. 15.6b similarly represents the relative velocities with respect to the observer, supposed to be in the middle row. The observed *radial* velocities should then be as shown in Fig. 15.6c. Stars in a particular direction will seem to approach, and those in the perpendicular direction to recede from, the observer. Actual observation bears out this peculiarity of radial velocities of distant stars. This constitutes an evidence in favour of the theory of rotation of the Galaxy. The position of the centre of the galactic system and the period of rotation for zones near the sun can be deduced from the effect of differential rotation discussed above. The period has been found with a fair degree of accuracy to be 250 million years. The position of the centre of the galactic system deduced is in fair agreement with results arrived at by other methods. We cannot do better than close the subject with a few remarks of Eddington:

"We may now sum up the evidence for the hypothesis of a rotation of the galaxy. An effect resembling differential rotation" —*i.e.*, the effect discussed by Oort.—"is observed in all classes of distant stars and also in the cosmic cloud pervading the system. These give consistent indications of the direction of the centre and they agree also as to the amount of differential rotation. The evidence from proper motions has small weight, but for what it is worth it supports that from spectroscopic radial velocities. * * * Our large orbital velocity of 200-300 km. per sec. is confirmed to some extent by observations of globular clusters and spiral nebulae which are too remote to pertake of it. * * * * Finally the very oblate shape of the stellar system is strongly suggestive of rapid rotation; and in the spiral nebulae, which are believed to be patterns of our galaxy, the rotation can be directly observed and measured."*

* *The Rotation of the Galaxy*, p. 18. (The Halley Lectures delivered on May 30, 1930).

APPENDIX A

A. 1. Determination of latitude and longitude of a place is essential in (1) map-making and (2) navigation. In map-making the relevant astronomical observations are made on land; but in navigation they have to be done at sea. On account of motion and rocking of ship observational methods employed on land are not available at sea. On land the transit instrument or the theodolite may be used; but at sea a different instrument called the *sextant* must be used in their place. Methods of observation must also be varied to suit the circumstance.

The principle of determination of latitude, in both cases, is the same:

(1) *By meridian observation:* The meridian altitude of the sun or a known star *i.e.*, of a star whose R.A. and Decl., are known, is observed; and the latitude is obtained from equation (5.1b). It is to be understood that z in the equation is the Z. D. corrected for refraction etc.

The meridian is not accurately known at sea. So in order to find the meridian altitude of a celestial body, observation of altitudes is begun a little time before the body comes to the meridian and is continued a little time after it has crossed it. The greatest altitude obtained is then taken to be the meridian altitude (SEC. 2.4). If two stars, one north and the other south of the zenith, be observed and the average of the latitudes deduced be taken, most of the errors of observation are either cancelled or considerably reduced. For, let z_1, δ_1 be the Z.D. and Decl., of the northern and z_2, δ_2 those of the southern star. Then the latitude of the place ϕ is given by $\phi = \delta_1 - z_1$ and $\phi = \delta_2 + z_2$ and the average value $= \dfrac{\delta_1 + \delta_2 + z_2 - z_1}{2}$ so that the errors of observation of z_1 is set against the errors of observation of z_2. Some of the errors, for example the error of the dip of the horizon, instrumental errors and personal error which are the same for both observations are therefore cancelled. Error of refraction is also considerably reduced.

(2) *By ex-meridian observation:* Let Z and P be the zenith and the pole of the observer and S a known star not on the meridian. Let z be the Z. D., corrected for refraction, of the star; and t the time of observation. Then in the triangle ZPS, $ZP = 90° - \varphi$, $ZS = z$, $SP = 90° - \delta$, $\angle ZPS = t - \alpha$, where α = the R.A. and δ = the Decl., of the star, and ϕ is the latitude of the place. We therefore have a relation between t and ϕ. If t be unknown, observe a second star and note the interval between the two observations. We thus have a second relation among the quanti-

ties mentioned. Solving the two equations both the latitude φ and t are obtained.

A. 2. The observation required is that of the altitude of a celestial body. And as already mentioned the instrument used at sea is the *sextant*.

We first prove a lemma which is useful in understanding the working principle of the sextant.

Lemma:—If θ be the angle by which a mirror is rotated, 2θ will be the angle by which the reflected ray from a fixed source is rotated.

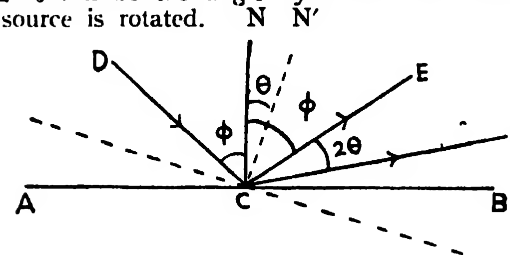

Fig. A.2a

Let AB be a mirror and CN the normal to it at C. Let the ray DC be reflected along CE; then $\angle$ DCN = $\angle$ ECN = φ, say. Now suppose the mirror to be rotated to the dotted position, through an angle θ. Let CN′ be the altered position of the normal and CF the altered direction of the reflected ray. Then $\angle$ NCN′ = θ, $\angle$ DCN′ = $\angle$ FCN′ = $\varphi+\theta$, $\angle$ FCN = $\varphi+2\theta$, $\angle$ ECN = φ. $\therefore \angle$ ECF = 2θ.

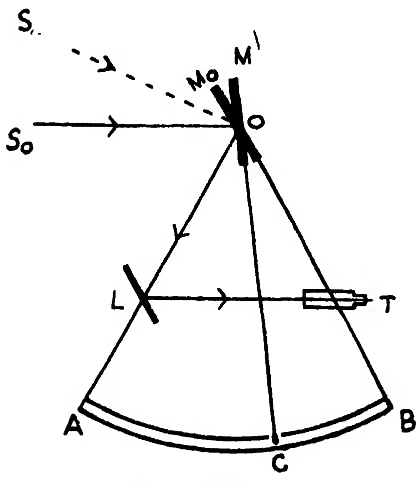

Fig. A.2b

The sextant consists of a graduated arc of a circle AB bounded by two arms OA and OB through the centre O; while a third arm OC is movable about the centre. A mirror L, the upper part of which is unsilvered is fixed to the arm OA; another mirror M is fixed to the movable arm OC at O. In the zero position of OC, the two mirrors are parallel. There is a telescope T fixed to the instrument so that an object can be viewed through the unsilvered part of the mirror L, at the same time that a ray incident along the arm OA is reflected into the telescope by the silvered part.

Suppose in the sextant the movable arm is at OC when the image of a star S is seen by double reflection at M and L to be coincident with a point of the horizon. Let S_0O be the horizontal direction. Consider a ray of light to proceed in the reverse di-

rection along LO; it will after reflection at L proceed along OS. We know that in the zero position of the arm the ray LO after reflection proceeds along OS_0.' Now $\angle SOS_0$ is the angle through which a particular reflected ray has rotated as the mirror is rotated from the zero position to the position OC. Hence $\angle SOS_0\theta = 2$ where θ is the angle through which the arm OC has been turned from the zero position.

Usually the graduations on the arc show double the actual angles. So the angle $\angle SOS_0$ which measures the altitude of the star S can be directly read on the arc. It may be noticed that the setting of the instrument for observation of altitude is not disturbed by any small displacement of the sextant in its own plane; and this fact makes it useful as an instrument for observation at sea.

A. 3. In order to *determine the longitude of a place,* we should know simultaneously (1) the local time and (2) the Greenwich time. For we know that if the longitude of a place west of Greenwich be L°, then (Greenwich time) – (local time) = $\frac{1}{15}$ L hours so that L is determined when the difference of the two times are known.

The first must be found by observation: in a fixed observatory the sidereal clock may be set by the transit of a known star; the local sidereal time can then be got from the clock. At sea the time of transit of a star must be inferred from the times when two altitudes of the star are equal; the time of transit is the time at the middle of the interval. It should be noted that while the meridian altitude is secured by observing the greatest altitude reached by a body, the time of transit cannot be obtained with precision by observing the time when the altitude is the greatest; for altitude remains the same near the maximum value for a considerable time. Ex-meridian observations may also be utilised, as explained in SEC. A. 1, because besides the latitude of the place of observation, the hour angle of the observed star and so the local sidereal time is obtained.

To find the Greenwich time we may

(1) carry chronometers set to Greenwich time on the ship or to the point of survey;

(2) establish, in case of land observation, electric connection with a place of known longitude for recording simultaneously the times of transit of a known star at both the places, and find the interval between the transits;

(3) depend on radio signals by which the errors and rates of the chronometers may be checked at short intervals.

Formerly the method of 'lunar distances' was used to find Greenwich time at sea. The movement of the moon among stars is fairly quick; and the distances of the moon from standard stars at different Greenwich times used to be recorded in the nautical almanac. By observing the distance of the moon from a suitable star Greenwich time might be obtained by interpolation in the

table. Certain complicated corrections had however to be applied to reduce the distance to one which would be observed from the centre of the earth, because the almanacs gave the distances as seen from the earth's centre. The method is not capable of any great accuracy and the publishing of lunar distances in the nautical almanac has been discontinued.

A. 4. Another method, known as *Sumner's method,* of finding the position at sea is as follows. Let the zenith distance z of a known star and the Greenwich time at the instant be observed. The position on the earth's surface where it would appear at the zenith is at once known. The position is called the *sub-stellar point:* if the sun be observed the position is called the *sub-solar point.* The ship is then on a small circle on the earth at a distance z from the sub-stellar point. Draw such an arc of a small circle on a map. Since the position of the ship is always roughly known, only a small line need be drawn. Let AB be such a line (Fig. A.4). A second line from a simultaneous observation of a second star, fixes the position of the ship by its intersection with the first. But since simultaneous observation is neither possible nor convenient, suppose the second observation is made after a known interval. An estimate of the distance travelled by the ship in the interval can be made by what is known as 'dead reckoning.' Draw A′B′ parallel to AB where EE′ is the distance travelled by the ship. The ship is then on A′B′. Let CD be the second Sumner line at the instant. The point of intersection, P, of the two Sumner lines is the position of the ship, and can be at once read from the map.

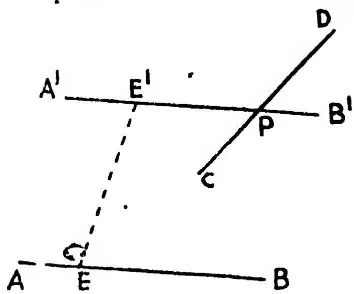

Fig. A. 4

APPENDIX B

Determination of the stationary position of a planet.

First let us consider the case of an inferior planet. Refer to Fig. 8.7 where S, V, E are the sun, the inferior planet and the earth respectively, when the planet is in inferior conjunction. Let E remain fixed and the planet occupy the relative position V_1 when stationary; the components of the velocities of the earth and the planet perpendicular to the line of sight EV_1 must be equal.

Let v_1 = velocity of the earth, v_2 = velocity of the planet r_1 = radius of the earth's orbit, r_2 = radius of the planet's orbit, $\varphi = \angle SEV_1$, the elongation of the planet when stationary, $\theta = \angle ESV_1$. If t be the interval between the inferior conjunction and the stationary phase and T the synodic period of the planet, then $\theta = 2\pi \times t/T$. We must have, $v_1 \cos\varphi = v_2 \cos(\theta+\varphi)$. But by equation (8.5),

$$v_1 \div \sqrt{\frac{1}{r_1}} = v_2 \div \sqrt{\frac{1}{r_2}}. \text{ Therefore } \frac{\cos\varphi}{\sqrt{r_1}} = \frac{\cos(\varphi+\theta)}{\sqrt{r_2}} \qquad \text{(Ba)}$$

Also, from the triangle ESV_1, $r_1/\sin(\theta+\varphi) = r_2/\sin\varphi$ (Bb)

Eliminating θ from the two equations, we have

$(r_2/r_1)\cos^2\varphi + (r_1^2/r_2^2)\sin^2\varphi = 1$

Or $(r_2/r_1)\cos^2\varphi + (r_1^2/r_2^2) - (r_1^2/r_2^2)\cos^2\varphi = 1$

whence $\cos^2\varphi = r_1(r_2+r_1)/(r_2^2+r_2 r_1+r_1^2)$

$$\text{and } \varphi = \cos^{-1}\sqrt{\frac{r_1(r_1+r_2)}{r_1^2+r_1 r_2+r_2^2}} = \sin^{-1}\sqrt{\frac{r_2^2}{r_1^2+r_1 r_2+r_2^2}} \qquad \text{(Bc)}$$

To find the angle θ we have from equation (Ba)

$$\cos(\theta+\varphi) = \sqrt{(r_2/r_1)}\cos\varphi = \sqrt{\frac{r_2}{r_1}}\sqrt{\frac{r_1(r_1+r_2)}{r_1^2+r_1 r_2+r_2^2}} \quad \text{from (Bc)}$$

$$\therefore \theta = \cos^{-1}\sqrt{\frac{r_2(r_1+r_2)}{r_1^2+r_1 r_2+r_2^2}} - \varphi$$

$$\text{Therefore } \cos\theta = \frac{\sqrt{r_2(r_1+r_2)}}{\sqrt{r_1^2+r_1 r_2+r_2^2}} \cdot \frac{\sqrt{r_1(r_1+r_2)}}{\sqrt{r_1^2+r_1 r_2+r_2^2}} + \frac{r_1 r_2}{r_1^2+r_1 r_2+r_2^2}$$

$$\text{Or } \cos\theta = \frac{\sqrt{r_1 r_2}}{r_1-\sqrt{r_1 r_2}+r_2} \quad i.e.\ \theta = \frac{2\pi t}{T}\cos^{-1}\frac{\sqrt{r_1 r_2}}{r_1-\sqrt{r_1 r_2}+r_2} \qquad \text{(Bd)}$$

The equation (B.c.) gives the elongation at the stationary position and the equation (Bd) gives the interval between the inferior conjunction and the stationary phase.

APPENDIX C

Calculation of the effect of collimation, level and deviation errors of the transit instrument.

First suppose only the collimation error of amount c is present. The false meridian is represented by the small circle in Fig. 5.5c. Let σ be the star on the false meridian. Draw σM perpendicular to the true meridian; $\sigma M = c$. The interval between the passages of the star over the true and false meridians is proportional to the hour angle σPM which $= \sigma M \div \cos\delta = c \sec\delta$ (compare sec 5.10) .... (C.a).

Next suppose only the level error of amount b is present. The position of the false meridian is as shown in Fig. 5.5e Let σ be the star on the false meridian. Draw $\sigma\sigma_1$ perpendicular to the true meridian. Since the arc ZZ_1 perpendicular to the false meridian $=$ b and $\sigma_1 Z = \varphi \ \delta$ where φ is the latitude of the place and δ the declination of the object $\sigma\sigma_1 / \cos(\phi - \delta) = b$. And as before the hour angle $\sigma P \sigma_1 = \sigma\sigma_1 \div \cos\delta = b \cos(\varphi - \delta) \sec\delta$ .... (C.b)

Lastly suppose only deviation error of amount k is present. The false meridian is represented in Fig. 5.5c. Let σ be the star on the false meridian. Draw $\sigma\sigma'$ perpendicular to the true meridian. (σ and σ' are not shown in the figure) Then as $Z\sigma'$ is equal to $\varphi - \delta$, $\sigma\sigma' / \sin(\varphi - \delta) = k$; and the hour angle $\sigma P \sigma' = \sigma\sigma' / \cos\delta = k \sin(\varphi - \delta) \sec\delta$ ... (C.c)

Since in practice these errors are very small, their joint effect may be obtained by adding them together. In each case, the correction we have found is to be added to the observed time of transit. Hence the total correction to be added to the observed time of transit in order to obtain the true time of transit is given by $t = c \sec\delta + b \cos(\varphi - \delta) \sec\delta + k \sin(\varphi - \delta) \sec\delta$ (C.d).

APPENDIX D

Astronomical Constants

(1) THE EARTH

Longer semi-diameter = 3963 miles
Shorter semi-diameter = 3950 miles
Mean density = 5.53
Mass = 5×10^{21} tons.

(2) THE SUN.

Diameter = 864,000 miles = 108 times the earth's diameter, approximately.
Mass = 329,300 times the earth's mass.
Mean denstiy = 1.42 approximately.
Distance from the earth = 92,900,000 miles approximately.
Diurnal parallalx = 8.″80.

(3) THE MOON

Diameter = 2160 miles approximately = a little more than one fourth of the earth's diameter.
Mass = 1/81 times the mass of the earth, approximately.
Mean density = 3.34.
Distance from the earth = Between 222,000 and 253,000 miles = 60 times the earth's radius, approximately.
Diurnal parallax = 57′ 2.″70.

(4) THE PLANETS.

Planet	Distance from the sun (in terms of the earth's distance).	Periodic time (in tropical years)	Semi diameter (in miles)	Number of Satellites
Mercury	.39	.24	1,504	nil
Venus	.72	.62	3,788	nil
Earth	1.00	1.00	3,963 3,950	1
Mars	1.52	1.88	2,108	2
Jupiter	5.20	11.86	44,350 41,390	9
Saturn	9.54	29.46	37,530 33,580	9 and 3 rings
Uranus	19.19	84.02	15,440	4
Neptune	30.07	104.79	16,470	1
Pluto	39.60	249.17	*	*

5. TIME

Length of the sidereal day = 23 hr. 56 min. 4.09 sec.
Sidereal month = 27.321661 days.
Synodic month = 29.530588 days.
Tropical year = 365.2422 mean solar days.
Sidereal year = 365.2564 mean solar days.
Anomalistic year = 365.2596 mean solar days.

(6) THE CELESTIAL SPHERE

Obliquity of the ecliptic = 23° 27′ 8″. 26 − 46″. 84T, where T is measured in centuries from 1900.

General precession = 50″. 256 + . 0022″ T, where T is measured in centuries from 1900.

Constant of aberration = 20″.47.

Constant of nutation = 9″.21.

Solar apex; R.A. = 18 hr. or 270°; Decl. = 34°.

Galactic pole; R.A. = 12 hr. 40 min. or 190°; Decl. 28°.

(7) DISTANCES OF A FEW NEAREST STARS

Star	Distance in light-year
α Centauri, A, B, C	4.28
Sirius A, B	8.6
Procyon A, B	11.1
61 Cygni A, B,	11.1

(8) MISCELLANEOUS

Constant of gravitation = 6.658×10^{-8} C.G.S. units.
Velocity of light = 186,270 miles per second.
1 mile = 1.609 km. 1 cm. = .3937 inch.
1 light-year = 6×10^{12} miles. 1 par sec = 3.26 light-years.

MISCELLANEOUS EXERCISES

1. Prove that when a star is on the meridian one-half of its visible path is accomplished.

2. Two stars have declinations which are equal but one north and one south. Show that the time one of them is above the horizon is equal to that of the other below the horizon. (ANNAMALAI 1938).

3. Show that for a place within the arctic or antarctic circle the points of intersection of the ecliptic with the horizon travel completely round the horizon during a sidereal day, but that for any other place they oscillate about the East and West points. (ANDHRA '42 Part III).

4. What is the lowest latitude in the arctic circle at which there is no twilight at mid-winter? (Answer: $84°32'$).

5. A and B are two places on the earth such that the latitude of A exceeds that of B by $18°$ and A lies in the frigid zone. Show that, if on any day A has the sun above the horizon continuously for 24 hr. B has twilight throughout night on that day. (ANDHRA '42 Part II).

6. Show that all places on the earth at which the sun rises at the same instant lie on a great circle. Show further that if δ be the declination of the sun on any day the inclination of the above great circle to the equator is $90° - \delta$ on that day.

7. If a and b be the azimuth and level errors of a meridian circle, show that, in the absence of any other error, the time of transit of a star in a latitude ϕ will be unaffected if its declination δ satisfies the relation

$$\delta - \phi = \tan^{-1} \frac{b}{a}$$

8. If a, b and c represent the azimuth, level and collimation errors of a meridian circle, show that the error in the time of transit of a star due to the above errors will be a minimum for a star whose declination δ satisfies the relation $c \sin\delta = a \cos\phi - b \sin\phi$, where ϕ is the latitude of the place of observation.

9. Show that a planet is in retrograde motion when it is nearest the earth. (ANNAMALAI 1938).

10. When a superior planet is stationary, show that its angular distance from the sun is

$$\pi - \tan^{-1} \frac{n}{\sqrt{n+1}}$$

when n is the ratio of radius of its orbit to that of the earth. (C.U. 1916).

11. If θ be the angle subtended at the earth by the sun and a stationary point of a planet's orbit and ϕ the greatest elongation of the planet, prove that

$$2 \cot \theta = \sec\frac{\phi}{2} + \operatorname{cosec}\frac{\phi}{2}.$$ (Patna Hon. '48)

12. The distance of a superior planet is n^2 times that of the earth, from the sun. Show that its retrograde motion lasts for $\frac{n^3}{\pi(n^3-1)} \cos^{-1} \frac{n}{n^2-n+1}$ of a terrestrial year. (ANDHRA '42 Part II).

13. Assuming the orbits of both to be circular and co-planar, prove that the geocentric motion of a superior planet will be direct when the square of the distance of the earth from the planet is greater than

$$\frac{(b^2-a^2)\,(b^{\frac{3}{2}}-a^{\frac{3}{2}})}{(b^{\frac{3}{2}}+a^{\frac{3}{2}})}$$

where a and b are the distances of the earth and the planet respectively from the sun. (ANNAMALAI 1933).

14. If α and α' be the semi-vertical angles of the cones of umbra and penumbra of the earth and ϕ the sun's apparent semi-diameter show that

$2 \sin \phi = \sin\alpha + \sin\alpha'$. (Patna Hon. '48).

15. If a total lunar eclipse occur at the summer solstice, and the moon is seen in the zenith at the middle of the eclipse, find the latitude of the place of observation. (Answer: 23°28′ S.)

16. If $f(r,p)=0$ be the equation to the earth's orbit, shew that $f\left(\frac{c^2}{p}, \frac{c^2}{r}\right)=0$ will be the equation to the path which a star appears to describe in consequence of aberration. (C.U., Hon. 1933).

17. If K be the constant of aberration, V the velocity of light, G the constant of gravitation, M the mass of the sun and r the radius of the earth's orbit, show that

$$GM = rK^2V^2.$$

18. If r and r′ (r being greater than r′) be the radii of the orbits of two planets round the sun, show that the ratio of the

relative velocities of the inner planet with respect to the outer at conjunction and opposition is

$$\frac{\sqrt{r}-\sqrt{r'}}{\sqrt{r}+\sqrt{r'}}$$

19. Prove that the aberration of a star situated at the first point of Aries is greatest at the equinoxes.

20. Show that when a planet is stationary its position is unaffected by aberration. (C.U. Hon. 1948).

21. If a, b and a′, b′ be the semi-major and semi-minor axes of the parallactic and aberrational ellipses respectively, prove that $ab'=a'b$

22. If at a place a star on the ecliptic has, at some time of the day, its displacement due to aberration along a vertical plane, show that the place must lie in the torrid zone.

23. If P be the distance of a star in light-years, p its parallax and k the coefficient of aberration, show that

$$2\pi Pp=k.$$

24. If l (l being greater than 66°32′) be the latitude of a star prove that its maximum and minimum declination due to precession are $180°-(l+\omega)$ and $l-\omega$ respectively where ω is the obliquity of the ecliptic.

25. If the proper motion of a star perpendicular to the line of sight be μ'' per year and p'' be its annual parallax then the velocity v of the star perpendicular to the line of sight is given by $v=\frac{\mu}{2\pi p}V$ where V is the velocity of the earth in its orbit.

ANSWERS TO EXERCISES

Exercise 1

4. No; yes; one. 5. No. 6. Sides: 180°-A, 180°-B, 180°-C; angles: 180°-a, 180°-b, 180°-c. 7. Less than 4 right angles. 8. Two diametrically opposite points.

Exercise 2

5 R.A. 100° and 280°.

		R.A.	Decl.	Latitude	Long
6.	21st March ...	0h	0°	0°	0
	21st June ...	6h	23°28′	0°	90
	23rd September ...	12h	0°	0°	180
	21st December ...	18h	−23°28′	0°	270

8. Rises at noon; Crosses the meridian at sun-set (i.e., 6 P.M.), 9. 0 hr. 12 min. A.M. (roughly); 11 hr. 48 min. P.M. (roughly). 10. 60°.

Exercise 3

(π is taken to be $= \frac{22}{7}$)

3. 3980 miles approximately. 4. 104.8 miles approximately. 5. .22 inches. 6. 30°. 7. 2.64 ft. (Distance of the top of the post from the earth's centre=4000 miles). 8. 698 miles approximately.

Exercise 4

1. (*i*) 38°28′ (*ii*) 38°28′. 2. 83°28′; 36°32′. 3. (*i*) No; (*ii*) Yes. 5. 45°. 6. 48°32′. 7. 65 days approximately (rate of change of declination is assumed uniform). 8. Latitude of the place=0° and declination of the star=0°. 9. 23°28′. 10. 47°15′49″; 39°27′15″. 11. 59°46′. 13. (*i*) 18hr.; 66°32′ (*ii*) 6hr.; 23°28′. 14. 89°26′. 15 10° and 20°. 17. (*i*) 10hr. 56min. 4sec. P.M.; (*ii*) 10hr. 1min. P.M. 18. 24hr. 20. Latitude of the place.

Exercise 5

3. $\frac{2}{15}$ sec; $\frac{2}{15}$ sec. 4. 2t. 5. $\frac{\sqrt{3}}{15}$ sec.

Exercise 6

2. 9° (taking cos 9°=.9876). 3. 50 miles nearly. 4. 44°58′53″ nearly. 5 z+43″.7 nearly.

6. $$1+\frac{\pi-(z_1+z_2+2\phi)}{\tan z_1+\tan z_2}$$ where all the angles are in radians.

Exercise 7

1. 865682 miles nearly. 2. 945 days nearly. 3. 1/60 nearly. 4. 18.5 miles per sec. 5. 28 days nearly.

Exercise 8

1. 329100 nearly. 2. d sec (78°45′) where d is the sun's distance from the earth. 3. 129.5 days nearly. 4. Venus' motion—retrograde, Jupiter's motion—direct. 5. $\frac{\sqrt{10}}{2}$. 6. .59 years or 215 days nearly; 526 days nearly. 7. 90°; $90° - \phi$. 8. 6/17. 9. 263 days nearly. 10. 1″.48; .71″ approximately. 11. (*i*) proportional to $\pi - \sin^{-1}(25/38)$; (*ii*) $\tan^{-1} 1.52$.

Exercise 9

3. About 244 days.

Exercise 10

2. Latitude−28°28′. 3. (*i*) $(2000/537)^2 n$; (*ii*) $(2000/537)^2 n . \cot^2 \theta/2$ where θ is the elongation of the Moon. 4. 21°32′; 68°28′. 5. 135/7 degrees. 8. 61°32′. 9. .07 approximately.

Exercise 11

1. 1°54′. 2. 16/57. 3. 92800000 miles correct to the first three significant figures. 4. 936585. miles nearly. 5. $\cos^{-1}(k/p)$. 7. 8 hr. 8. .26 approximately.

Exercise 12

1. 20952 miles nearly. 2. (*i*) 550/861 *i.e.*, 16/25 miles per sec. nearly; (*ii*) $\frac{1}{3}$ miles per sec. nearly. 3. parallax of the moon+parallax of the sun−angular radius of the sun; the numerical value varies according to the values of the different quantities. 5. 972 miles per hour nearly.

Exercise 13

1. 5 min. 20 sec. P.M.; 5 hr. 9 min. 20 sec. A.M. 2. 36°30′ west longitude. 3. 6 hr. 47 min. 12 sec. 4. +2 min. 30 sec. 5. 1 day should be subtracted after 3600 years. 6. 3 hr. 38 min. 38 sec. nearly. 7. 25th October. 8. 7 hr. 18 min. 48 sec. A.M.

Exercise 14

1. (*i*) A small circle of polar radius=$\sin^{-1}(\alpha/k)$ where α is the displacement and k is the coefficient of aberration; (*ii*) two. 2. Maximum on June 21 and December 21; Minimum on March 21 and September 23. 3 Aberration is maximum on June 21 and December 21; minimum on March 21 and September 23. Parallax is maximum on March 21 and September 23; minimum on June 21 and December 21. 4. 0.32 sec. approximately; 11 : 756. 5. 1/p parsecs. 6. $(19.34 \times 10^{12})/p$ miles nearly.

UNIVERSITY QUESTION PAPERS

Calcutta University

SYLLABUS OF PASS COURSE

The subject is to be treated mathematically but without the use of spherical trigonometry

1. The earth. 2. Astronomical co-ordinates. 3. Astronomical clock, Transit Instrument, Meridian Circle and Equatorial. 4. Atmospheric Refraction. 5. The Sun and the Solar System. 6. Parallax. 7. Determination of the First Point of Aries. 8. Precession, Nutation and Aberration. 9. The Moon. 10. Lunar and Solar eclipses. 11. Measurement of time. 12. Determination of Latitude and Longitude by simple methods. 13. The fixed Stars.

SYLLABUS OF HONOURS COURSE

THEORETICAL :—The subject of the Pass Course treated more fully. (*Candidates will be expected to possess an elementary knowledge of spherical trigonometry and to apply it to the discussion of simple problems in Astronomy.*)

PRACTICAL :—The students should be required to make observations with a view to—

(1) the determination of Latitude; (2) the determination of Time; (3) the determination of Longitude; (4) the determination of Azimuth; (5) the use of methods suitable at sea; and (6) the plotting of the apparent path of one planet among the stars.

PASS PAPERS

1944.

1. (a) Define (i) prime vertical (ii) zenith distance (iii) right ascension (iv) hour angle, of a heavenly body.

What is the hour angle of the sun at sun-rise on the 21st March? What is the highest latitude, north or south, at which it is possible to see the sun in the zenith at noon?

(b) What are the equinoxes? What is the equinoctial colure? How would you distinguish a fixed star from a planet?

2. Explain the phenomenon of twilight. On what factors does its duration depend? Why does the duration in the same place vary according to the season of the year? see page - 46!

Can twilight last all night at Calcutta? Give reasons for your answer.

3. (a) What is the geocentric parallax of a heavenly body? Show that it varies as the sine of the apparent zenith distance of the body.

(b) Compare the effects of diurnal parallax with those of atmospheric refraction.

4. What are ecliptic limits? Determine the greatest and least number of eclipses possible in a year.

Find approximately the maximum duration of an eclipse of the moon.

5. What is the equation of time?

Assuming the greatest equation of time due to obliquity to be greater than that due to eccentricity, prove that the equation of time vanishes four times a year.

How many times a year would it vanish were the magnitudes of the maxima reversed?

6. (a) Define—(i) Civil year (ii) Tropical year (iii) Anomalistic year.

(b) What is the Julian calendar? Explain what is meant by the Gregorian correction.

(c) How would you reduce a given interval of mean time to sidereal time?

7. Assuming that on a certain day at Greenwich the Right Ascension of the mean sun was 10 hours at 12 o'clock, find for a place whose longitude is 60° west, the time by the ordinary clock on the same day, when the time by an astronomical clock at the place was 14 hours. (Correction for Greenwich time being 1 min. 18 sec. due to the change in mean sun's R.A.)

1945

1. Define—celestial meridian, prime vertical, celestial latitude, hour angle.

Explain Foucault's pendulum experiment to prove the earth's rotation.

2. Describe a transit instrument and define accurately its line of collimation.

A circumpolar star crosses the meridian at altitudes 10° 11′ 17″ and 72° 15′ 31″; find the latitude of the place and the star's polar distance.

3. What is the cause of twilight?

Does twilight ever last all night at Paris (latitude 48° 50′)? Give reason for your answer.

Prove that the duration of twilight depends upon the latitude of the place and the declination of the sun.

Find the lowest latitude at which it is possible to have a mid-night sun.

4. Prove Bradley's formula for the coefficient of refraction, and state accurately what observations have to be made in applying it.

Give a direct explnation of the effect of aberration on the position of stars and indicate on the celestial sphere the point towards which they are displaced.

5. State the causes of a solar eclipse, and explain under what circumstances it is (a) total (b) partial or (c) annular.

What are the causes of a lunar eclipse? Why does not the phenomenon occur at every full moon? How are the lunar ecliptic limits found?

6. Define the "equation of time" and state from what causes it rises. How does its magnitude vary throughout the year? What is its greatest value? How many times a year does it vanish and on what dates?

Express in mean time an interval of 12 hr. 16 min. 26 sec.

1946.

1. Define—ecliptic, celestial latitude, terrestrial latitude, zodiac. Prove that the altitude of the celestial pole at any place is equal to the latitude of the place. Prove that the altitude of a star is greatest when on the meridian.

What is the hour angle of the sun at sun-rise on the 21st. March?

2. Give a proof of the earth's rotation from the experiment of letting a body fall from the top of a high tower. How do you eliminate the error of eccentricity of the meridian circle? Find the zenith point of the meridian circle.

3. Find the co-efficient of refraction by Bradley's method. What is the advantage of this method, and what are its disadvantages? Prove that the effect of parallax on a heavenly body is to depress it in the heavens The sun's horizontal parallax is 8″.8, find the true zenith distance corresponding to an observed zenith distance of 60°.

4. Explain the phenomenon of seasons. Explain the phenomenon of Harvest moon.

5. Describe Flamsteed's method of finding the Right Ascension of a star. What are its advantages?

State the causes to which the equation of time is due. Given that the sun rose on a certain date at 6 hr. 56 min. and set at 4 hr. 32 min.; find the equation of time.

6. Find the minimum and maximum number of eclipses of the sun and the moon in a year.

Find the duration of twilight at the equator during the equinoxes.

Find the latitude of the place for which twilight just lasts all night when the sun's declination is 16°N.

1947.

1. Define—ecliptic, Zodiac, Right Ascension, Celestial latitude.

Give proof of the earth's rotation from the experiment of letting a body fall from a high tower.

2. Describe the transit instrument and explain how it is corrected for the various errors to which its readings are subject.

How do you find the pole by observation of the transit of a circum-polar star?

3. Enumerate the different causes which produce an apparent change in the position of a body, and explain the change produced in each case.

Prove that, owing to aberration, a star situated at the pole of the ecliptic will in the course of a year appear to revolve round its true position in a circle.

4. Explain the phenomenon of the seasons.

Explain what is meant by twilight, and prove that the duration of twilight depends on the latitude of the place and the declination of the sun.

5. Describe Flamsteed's method of finding the Right Ascension of a star. What are its advantages? Discuss the effect of precession upon the result.

6. State the causes of a solar eclipse and explain under what circumstances it is (a) total, (b) partial and (c) annular.

State the causes to which the equation of time is due; and establish—length of afternoon minus length of morning = twice the equation of time.

1948.

1. Define: Azimuth, Right ascension, Hour angle and Celestial latitude.

Give a proof of the earth's rotation from the experiment of letting a body fall from the top of a high tower.

2. Find the coefficient of refraction by Bradley's method. What is the advantage of this method and what is its disadvantage?

The apparent zenith distance of a star at lower and upper culminations are 75° 3′ 13.2″ and 1°53′ 18.6″ south; the amounts of refraction in the two observations are 3′ 41.9″ and 1.9″ respectively. Find the latitude of the place and declination of the star.

3. Prove that the velocities of two planets round the sun are inversely as the square roots of their distances from the sun.

The greatest and the least apparent diameter of the sun are 32′ 36″ and 31′ 32″ respectively; show that the eccentricity of the earth's orbit is approximately 1/60.

4. Define 'diurnal parallax' of a celestial body. Find the amount of diurnal parallax of a celestial object whose zenith distance is given.

The sun's horizontal parallax is 8.8″. If its observed zenith distance be 60°, find its true zenith distance.

5. Calculate the conditions for (a) a lunar, and (b) a solar eclipse.

How are solar and lunar ecliptic limits found?

6. What is meant by the 'equation of time'?

Prove by reference to its causes that the equation of time vanishes four times a year.

The longitude of Paris is 2°20′E, and the longitude of Dublin is 6°40′W. Find the local time at Dublin when it is 12h. 6m. at Paris.

Patna University

1947 (Pass)

1. (*a*) Define the term Right Ascension, Azimuth, Declination and Celestial Longitude, and represent them in a figure.

(*b*) At the summer solstice the meridian altitude of the sun is 75°. What is the latitude of the place? What will be the meridian altitude of the sun at the Winter solstice?

2. (*a*) Enumerate the errors to which a Transit Instrument is liable, and explain how the deviation error may be detected.

(*b*) The zenith distances of a star at lower and upper culminations are found, after correcting for refraction, etc., to be 76°4′ and 2°52′S. respectively. Find the latitude of the place.

3. (*a*) Find the duration of twilight at the equator during the equinoxes.

(*b*) Prove that the apparent motion of Mars is retrograde when we are closest to it and direct when we are farthest from it.

4. (*a*) Prove that the parallax of a heavenly body varies as the sine of its apparent zenith distance.

(*b*) The observed altitude of a star is found to be an angle whose sine is 3/5; calculate the true position of the star, given that the co-efficient of atmospheric refraction is 58.″2.

5. (*a*) Explain the occurrence of a solar eclipse and discuss the circumstances under which it is (i) total, (ii) partial, and (iii) annular.

(*b*) Convert 22h. 26m. 1s. sidereal time into mean solar time, being given the R.A. of the mean sun at mean noon as 20h. 4m. 17s.

1948 (Pass)

1. Describe the theory of Foucault's Pendulum Experiment for proving the earth's rotation, and give a short account of his experiment.

Why is the sun never seen in the zenith at Patna (Lat. 25°30′N.)?

2. Indicate Bradley's or any other method for determining the co-efficient of astronomical refraction.

An altitude of a star is observed and found to be the angle whose sine is 5/13. Calculate the true position of the star, assuming the amount of refraction at an altitude of 45° to be 58.2″.

3. (*a*) Explain the direct and retrograde motions of superior and inferior planets.

(*b*) Venus is an evening star and is stationary. Find which way she begins to move.

4. Indicate a method for obtaining the annual parallax of Jupiter.

If p be the number of seconds in the annual parallax of a fixed star, show that the time taken by light to reach us from this star is approximately, 16/5p years. [velocity of light =186,000 miles per second.]

5. Prove that the length of afternoon minus the length of morning is equal to twice the equation of time.

Given Mean time at Greenwich 10h. 30m. 40s., convert it into sidereal time at the same place if the Right Ascension of the sun at the previous Greenwich mean noon is 18h. 12m. 35s.

1947 (Supplementary Pass)

1. (*a*) Show how the latitude of a place can be determined by observing the meridian altitude of a star of known declination.

(*b*) What is meant by a circumpolar star? What is the limit of declination for stars which are circumpolar in latitude 60°N.?

2. (*a*) Define the synodic period of a planet, and show how to find its Periodic time when the synodic period is known.

(*b*) The greatest and least apparent angular diameter of the sun are 32' 36" and 31' 32". Calculate the eccentricity of the earth's orbit.

3. (*a*) Show that the atmospheric refraction of a heavenly body varies as the tangent of the apparent zenith distance, temperature and pressure being constant.

(*b*) Assuming the horizontal parallax of the moon to be 1/60, and her apparent diameter to be 1,963", find the moon's diameter in miles, the earth's radius being 4,000 miles.

4. (*a*) Prove that the least possible number of eclipses in a year is two, both of the sun.

(*b*) Find the declination of the sun when twilight begins to last all night at Dublin (lat. 53°20').

5. (*a*) Define equation of time, and prove that (length of afternoon)—(length of morning) = 2 (equation of time).

(*b*) Find the sidereal time corresponding to 8h. 15m. 40s. P.M. on Dec. 20, given that the sidereal time of mean noon was 17h. 55m. 8s.

Andhra University

1940.

1. (*a*) Define the terms—Prime vertical, Solstices, Azimuth.

(*b*) Find the meridian altitudes of the sun at summer and winter solstices at Madras (lat. 13°4' North).

(*c*) The R.A. and Decl. of a star are 3h. 2m. and 40°38'N. At what time approximately will it cross the meridian of Madras on 9th November?

2. (*a*) Explain the phenomenon of twilight and find its duration at the equator during the equinox.

(*b*) Find the lowest latitude at which it is possible for twilight to last all night.

3. Describe the transit instrument and enumerate its possible errors. Show how to use it for determining the R.A. of a celestial body.

4. (*a*) Outline a method of determining the position of the first point of Aries.

(*b*) Find approximately at what times on 1st March the ecliptic will pass through the East and West points of Madras? What are its inclinations to the horizon then?

5. (*a*) State Kepler's Laws of planetary motion.

(*b*) The apparent diameter of the sun when least is 31′ 32″ and when greatest is 32′ 35″. Calculate the eccentricity of the earth's orbit round the sun.

6. Explain the phenomenon of Astronomical refraction. Indicate with reasons, the effect of refraction on (1) the apparent position of a celestial body (2) the times of rising and setting of the sun (3) the apparent shape of the full moon when rising or setting.

7. (*a*) Describe the method of finding the longitude of a place.

(*b*) A chronometer is set by the standard clock at Greenwich at 6 A.M. It is then taken to a place A and indicates noon when the local time is 11hr. 49m. 50s. The chronometer is then brought back to Greenwich on the same day and indicates 9 P.M. when the correct time is 8h. 59m. 55s. Find the longitude of A, assuming the rate of error of the chronometer to be uniform.

8. (*a*) What is meant by the synodic period of the Moon's nodes? Find its duration, given that the moon's nodes have a retrograde motion of 19°21′ per annum.

(*b*) Explain how sometimes three eclipses occur within a month.

9. Account for the following :—

(*a*) Venus exhibits all the phases presented by the Moon, but Mars is always either full or gibbous.

(*b*) The sidereal clock is not fit for civil purposes.

(*c*) The moon always presents practically the same face to the earth.

(*d*) A Solar eclipse is visible only for a small portion of the earth, whereas a lunar eclipse is visible for more than half the earth at the same time.

1944.

1. (*a*) Show that the latitude of a place is equal to the altitude of the celestial pole.

(*b*) Show with the aid of a diagram, that the sum of the hour angle of a star and its R.A. is equal to the local sidereal time.

(*c*) Find the altitude of the star Capella (Decl.=45°56′N) at lower transit if at upper transit the star is in the zenith of the place. At what latitude is the star just circumpolar?

2. (*a*) Describe the equatorial and mention the uses of the instrument.

(*b*) The observed times (by a sidereal clock) of consecutive transits of a star whose R.A. is 13h. 51m. 14.3s. are 13h. 50m. 36.5s. and 13h. 50m. 37.1s. Find the rate of the clock.

3. (*a*) If α, δ be the Sun's R.A. and Decl., λ his longitude and ω the obliquity of the ecliptic prove that

$$\cos\lambda=\cos\alpha\cos\delta \text{ and } \tan\lambda=\tan\alpha\sec\omega.$$

(*b*) Find the latitudes at which ecliptic is perpendicular to the horizon at some time during the day. What is then the local sidereal time?

4. (*a*) Explain what is meant by the equation of time and show that it vanishes 4 times a year.

(*b*) The equation of time on a certain day was −4m. 30s. and the sun rose at 5h. 21m. 30s. on that day. Find the time of sun-set.

5. (*a*) Compare the effects of Geocentric parallax and Astronomical Refraction on the position of a heavenly body.

(b) At meridian transit in latitude 51°31′N the observed Z.D. of the moon's upper limb is 35°20′45″, the moon's semi-diameter is 16′9″ and the horizontal parallax is 59′16″. Taking refraction into account find the moon's declination. (Take the co-efficient of refraction=58″.2).

6. (*a*) State Kepler's laws of planetary motion and show how the third law enables the determination of the mass of a planet which possesses a satellite.

(*b*) The synodic period of Mars is 780 days and the radius of its orbit (assumed circular) is 1.52 times the radius of the earth's orbit. Calculate (*i*) its phase at quadrature (*ii*) its elongation 195 days before opposition.

7. (*a*) Explain the terms—lunation, metonic cycle, age of the moon, moon's librations.

(*b*) The moon is observed at a certain instant to be on the meridian, 3/4 full, and with the bright limb to the west. Find the age of the moon and the approximate time of the day.

8. (*a*) Explain what is meant by the constant of aberration and describe a method of finding it by observation. What is the relation between this constant and the solar parallax?

(*b*) Show that a star lying on the great circle joining the poles of the equator and the ecliptic has no precession in declination.

9. (*a*) Name five stars of the first magnitude and the constellation in which they lie.

(*b*) Write short notes on: Saros, Transit of Mercury, Standard time.

Agra University

1942.

1. (*a*) Represent very neatly and carefully on one diagram the celestial equator, horizon, ecliptic, the latitude, declination, right ascension, hour angle and azimuth of a star.

(*b*) Under what conditions would the azimuth of a star remain constant from rising to transit?

2. Prove the following statement, connecting the mean time at a given meridian with the corresponding sidereal time :—

Sidereal time=mean time+mean sun's R.A.

Assuming that on a certain day at Greenwich the R.A. of the mean sun was 10hr. at 12 O' clock find, for a place whose longitude is 60° west, the time by an ordinary clock on the same day, when the time by an astronomical clock at the place was 14hr. You may assume that a sidereal day contains 23h. 55m. 4s. mean time.

3. Define the terms 'lunation' and the cycle of Meton.

Verify the following statement :—

'At the end of 19 years the sun and the moon return to the same relative position with regard to the fixed stars and the full moons fall again on the same days of the month and only one hour sooner'.

1943.

1. (*a*) What is twilight? Find the conditions for twilight to last over all night.

(*b*) Find the duration of twilight at the equator during the equinoxes.

2. (*a*) State Kepler's laws of motion.

Deduce the third law of Kepler from the gravitational law.

(*b*) Prove that the velocities of any two planets are connected with their distances from the sun by the relation

$$\frac{v}{v'}=\frac{\sqrt{r'}}{\sqrt{r}}$$

3. (*a*) What is the equation of time? Show that it vanishes four times a year.

(*b*) Find the mean solar time at Madras (longitude=80°14′19″E) corresponding to apparent time 8 P.M. there on September 6th, 1893, being given the following from the 'Nautical Almanac' for 1893.

At Greenwich mean noon

Equation of time on September 6th=−1 min. 52.42 sec.
Equation of time on September 7th=−2 min. 12.42 sec.

Dacca University

1935.

1. Define the terms : equinoxes, solstices, equinoctial and solstitial points, equinoctial and solstitial colures.

What are the dates of the equinoxes and solstices, and what are the corresponding values of the sun's declination, longitude and right ascension.

2. Explain the cause of the seasons. What would be their character if the earth's axis were at right angles to the ecliptic.

3. What is a circumpolar star? Show that the altitude of a circumpolar star is greatest or least when the star is on the meridian.

Define the latitude of a place and show how it can be determined by the transits of a circumpolar star over the meridian.

4. What do you understand by the annual aberration of a star? Prove the following :

(i) Annual aberration produces a displacement in the apparent position of a star towards a point on the ecliptic 90° behind the sun.

(ii) The amount of displacement varies as the sine of the earth's way.

(iii) A star, situated at the pole of the ecliptic, will, in the course of a year, appear to revolve round its true position in a circle whose angular radius is 20.49″.

5. Define sidereal time, mean solar time, equation of time.

Draw a diagram of the celestial sphere, and show on it the angles which measure sidereal time, mean solar time, the R.A. of the mean sun, apparent solar time and the equation of time.

1936.

1. Define the terms : Celestial pole, Celestial meridian, Celestial latitude, Celestial longitude and Hour angle.

Find the hour angle of the sun at sun-rise on 21st March.

2. Prove the formula for atmospheric refraction.

Show how the constant of refraction can be determined by the observation of a circumpolar star, when the latitude of the place is known.

3. What is twilight? Find the condition that twilight may last all night. Can twilight last all night at Dacca?

4. Define the terms : annual parallax, diurnal parallax and horizontal parallax.

Describe a method of determining the annual parallax of a star.

5. Define 'the solar and lunar ecliptic limits'.

Find the least possible number and the greatest possible number of eclipses during a year.

Calcutta University Honours Papers

1946.

1. Describe a transit instrument and define accurately its line of collimation.

State the conditions which must be fulfilled in order that a transit instrument may be in perfect adjustment.

In the case of Deviation error will the method of correction with the help of circumpolar stars be available everywhere? If not, why not?

If the western pivot of a transit instrument be α'' higher and β'' more to the north than the eastern, show that a star is unaffected whose

$$\text{N.P.D. is} = \text{Colat} + \tan^{-1}\left(\frac{\tan\alpha}{\tan\beta}\right)$$

2. (*a*) Describe in detail Flamsteed's method for finding the First point of Aries. What are the advantages of this method? Why is it that we can use the observed Z.D. in the final form of the result in this connection uncorrected for refraction?

(*b*) Find the angle between the ecliptic and the equator in order that there should be no temperate zone.

3. (*a*) Define Geo-centric Latitude of a place.

Show that the difference between geo-centric and geographical latitude of a place is approximately equal to $\frac{1}{2}e^2 \sin 2l$ where l is the geographical latitude of the place.

(*b*) A railway train is moving north-east at 40 miles an hour in latitude 60°. Find approximately, in numbers, the rate at which it is changing its longitude.

4. (*a*) Define the equation of time. Show that the equation of time vanishes four times a year.

(*b*) Find the R.A. of the sun at true noon on October 8, 1891 given that the equation of time for that day is -12m. 24s. and that the sidereal time of mean noon on March 21 was 23h. 54m. 52s.

5. (*a*) What are the effects of atmospherical refraction on the rising and setting of a celestial body?

Shew that at the Earth's equator at an equinox the time of sun-rise is accelerated by about 2m. 12s. owing to refraction.

(*b*) Show that twilight lasts all night about midsummer at any place whose latitude is not less than $48\frac{1}{2}°$. Find the duration of twilight at the equator at an equinox.

6. (*a*) Draw a neat sketch and find the limits of the Moon's geocentric position consistent with a Solar or Lunar eclipse. Explain with the help of your diagram why, on the whole, solar eclipses are more frequent than lunar.

(*b*) Supposing the centres of the Earth, Moon and Sun to be in a straight line and the Moon's and Sun's semi-diameters to be exactly 17′ and

16′, find the angular radii of the circles on the Earth over which the eclipse of the Sun is total taking the relative horizontal parallax as 57′.

1947.

1. (*a*) Describe the Equatorial and explain the adjustments and principal uses of the instrument.

(*b*) Can the clockwork by which the Equatorial is driven be regulated by an ordinary pendulum? Give reasons for your answer and explain how it can be properly regulated

(*c*) What is a micrometer? How would you test the zero reading of the position circle?

2. (*a*) Show that the refraction of a heavenly body, the temperature and pressure being constant, varies as the tangent of the apparent zenith distance. Is the law true for all zenith distances? If not, what is the approximate limit?

(*b*) Find the latitude of a place at which the observed meridian zenith distances of a circumpolar sta[illegible] are [illegible]8′ and 22°18′, given that the tangents of these angles are 1.09 and .41 respectively. Take the coefficient of refraction to be 58″.2.

3. (a) Which is the nearest of the superior planets to Earth? It is said that two satellites of this planet were discovered on September 5, 1877. Why was that date particularly favourable for observing the planet? State any striking peculiarity, you know of, about one of these satellites.

(*b*) What is the Synodic Period of a superior planet? If there are 378 days between two successive oppositions of Saturn, find the length of Saturn's year.

4. (*a*) Determine the Annual Parallax of a star by Bessel's Method.

Where must a star be situated so as to have no displacement due to parallax?

(*b*) The interval between eastern and western quadratures of Jupiter is 175 days and between two oppositions 400 days: find the annual parallax of the planet.

5. (*a*) Explain, how it is that we see more than half the Moon's surface. Explain the terms—node, phase, libration in connetion with the Moon.

(*b*) How is the distance of the Moon determined by observations made in the plane of the meridian? Why cannot the Sun's parallax be accurately determined in this way?

6. (*a*) Account for the phenomenon of a Lunar Eclipse. Show that it begins and ends at the same instant at all places from which it is visible.

What is the greatest number of lunar eclipses which can occur in a year? Give reasons for your answer.

(*b*) Find roughly the maximum duration of an eclipse of the Moon and the maximum duration of totality.

7. (*a*) Show that the equation of time vanishes four times a year.

(*b*) Find the (local) sidereal time at New York at 9h. 25m. 31s. (local mean time) on the morning of September 1, 1891.

Given, Longitude of New York=74°W.

Sidereal time of mean noon at Greenwich, Sept. 1=10h. 42m. 24s.

1948.

1. Describe a Transit Instrument. What is the line of collimation?

When is the instrument said to be in perfect adjustment?

How would you determine the Deviation Error by observing the transits of two known stars?

What is the advantage of chosing two stars whose right ascensions are not widely different in the above method?

2. Describe Flamsteed's Method for finding the First point of Aries.

What are the advantages of this method? Explain them.

What correction would you make in using this method for precession?

3. Define Heliocentric Latitude and Longitude of a celestial body.

Show that the Earth's heliocentric longitude differs from the sun's geocentric longitude by 180°.

Prove that in the course of the year the sun is as long above the horizon at any place as below it.

Explain how it is that winter is colder than summer although the sun is nearer.

4. Define "Dynamical Mean Sun" and "Mean Sun".

Discuss in some detail the causes of the equation of time. Obtain the relation between the lengths of morning and afternoon in terms of the equation of time.

On a certain date the sun-dial is 16m. 20s. before the clock. Given that the sun rose at 6h. 54m. find the time of sun-set.

5. Show that there may be as many as seven eclipses in a year namely either five solar and two lunar or four solar and three lunar under most favourable circumstances

Why is the Moon seen throughout a total eclipse?

What is the saros of the chaldeans?

6. Compare the corrections for aberration and annual parallax. How must a star be situated so as to have no displacement due to (*a*) aberration, (*b*) parallax?

Obtain a relation between the co-efficient of aberration and the equation of light.

Show that when a planet is stationary its position is unaffected by aberration.